Evaluation of electromotive force in interplanetary space

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Abstract. Electromotive force plays a central role in the turbulent dynamo mechanism and carries important information on the nature of the turbulent fields. In this study, an analysis method is developed for the electromotive force and the transport coefficients such as those for the \( \alpha \) effect (coefficient \( \alpha \)) and the turbulent diffusivity (coefficient \( \beta \)). The method is applied to a magnetic cloud event observed by the Helios 2 spacecraft in the inner heliosphere. The electromotive force is enhanced together with the magnetic cloud event by 1 or 2 orders of magnitude, suggesting that the magnetic field can locally be amplified in the heliosphere, presumably for a short time.

Keywords. Interplanetary physics (interplanetary magnetic fields) – space plasma physics (transport processes)

1 Introduction

The essential part in the dynamo mechanism amplifying the large-scale magnetic field lies in the existence of electromotive force. In the theory of mean-field electrodynamics, the electromotive force is defined as a statistically averaged vectorial quantity, and it is a cross product between the fluctuating flow velocity \( \delta U \) and the fluctuating magnetic field \( \delta B \),

\[
\mathbf{E}_\text{em} = \langle \delta \mathbf{U} \times \delta \mathbf{B} \rangle .
\]

Here, the angular brackets \( \langle \cdots \rangle \) denote the operation of ensemble averaging over many realizations. The electromotive force has dimensions of the electric field.

In the mean-field dynamo theory, the electromotive force is assumed to be linear in the mean magnetic field \( \mathbf{B}_0 \) to the first order, and also in the spatial gradient of the mean field such as the curl of the mean magnetic field, \( \nabla \times \mathbf{B}_0 \) (one may also use the current density \( \mathbf{j} \) using Ampère’s law), and the curl of the mean flow velocity, \( \nabla \times \mathbf{U}_0 \) (which is the vorticity). A more comprehensive form for the electromotive force using up to the first-order derivatives of the mean fields is expressed as (Yokoi, 2013)

\[
\mathbf{E}_\text{em} = \alpha \mathbf{B}_0 - \beta \nabla \times \mathbf{B}_0 + \gamma \nabla \times \mathbf{U}_0, \tag{2}
\]

where the first term on the right-hand side in Eq. (2) with the coefficient \( \alpha \) and the mean magnetic field serves as the amplification of the mean magnetic field by small-scale twisting flow motions. The second term with the coefficient \( \beta \) (which has the same dimension as that of the magnetic diffusivity) and the curl of the mean magnetic field (which is proportional to the electric current density for the mean magnetic field) is the turbulent magnetic diffusion, and the third term with the coefficient \( \gamma \) and the curl of the large-scale flow velocity the cross helicity dynamo.

A simpler form or a minimal expression of the electromotive force for a dynamo mechanism is composed of the two terms associated with the mean magnetic field only (Parker, 1955; Steenbeck et al., 1966; Steenbeck and Krause, 1966):

\[
\mathbf{E}_\text{em} = \alpha \mathbf{B}_0 - \beta \nabla \times \mathbf{B}_0. \tag{3}
\]

Equation (3) was historically proposed before the importance of the cross helicity term was recognized in studies on the sunspot number variation and the accretion disk dynamo (Yoshizawa, 1990; Yoshizawa and Yokoi, 1993; Yokoi, 1996, 1999). Here again, the first term with \( \alpha \) is responsible for the growth of the mean magnetic field, and it is essential to the \( \alpha \) effect due to the amplification of the magnetic field by a turbulent twisted flow motion. The second term with \( \beta \) is destructive and contributes to an effective diffusion of the large-scale fields when coupled to the induction equation (turbulent diffusion). Equation (3) is obtained from the...
induction equation for the fluctuating magnetic field using the first-order smoothing approximation (see, e.g., Choudhuri, 1998, for a concise derivation). The coefficients $\alpha$ and $\beta$ are in fact tensors (of rank 2 and 3, respectively). In the isotropic turbulence treatment, these tensors reduce to scalars (the $\alpha$ tensor becomes diagonal and the $\beta$ tensor is proportional to the Levi-Civita antisymmetric tensor). In our work, we use the simpler form (Eq. 3) for the sake of brevity in mathematical operations in the spirit of proof of concept.

It is important to note that the electromotive force is a second-order fluctuation quantity and is in the same class as the energy densities of the fluctuating fields (for the magnetic field and the flow velocity) and the helicity densities (magnetic helicity, kinetic helicity, and cross helicity). The electromotive force appears as off-diagonal elements of the covariance matrix composed of the fluctuating magnetic field and fluctuating velocity (Narita, 2017).

Here we propose an analysis method to determine the electromotive force using in situ spacecraft data in space plasma. While a large number of studies on turbulent space plasma (e.g., solar wind turbulence) concentrate on the behavior of the energy and helicity quantities (see, e.g., reviews and monographs such as Tu and Marsch, 1995; Petrosyan et al., 2010; Bruno and Carbone, 2013), only few observational studies have been performed related to the discussion on the electromotive force in space plasma physics, e.g., an impulsive solar wind event in the Earth magnetosphere (Lundin et al., 2003) and turbulent solar wind plasma (Marsch and Tu, 1992, 1993). In particular, Marsch and Tu (1992, 1993) discovered evidence that the electromotive force follows a power-law spectrum in the solar wind (indicating turbulent electromotive force) and is not simply proportional in the mean magnetic field. The electromotive force is, in contrast to the limited space plasma studies, one of the primary study targets in the laboratory pinch plasmas (Fontana et al., 2000; Ji and Prager, 2002) and in the liquid sodium experiment (Rahbarnia et al., 2012). This paper is motivated by a wish to fill the gap in the application of the electromotive force between laboratory and space plasma studies.

2 Estimates of transport coefficients

When modeling with $B_0$ and $\nabla \times B_0$ and neglecting the other terms or contributions from the cross helicity or the higher-order derivatives of the mean fields, it is possible to evaluate the transport coefficients $\alpha$ and $\beta$ directly from the measurement of the electromotive force. To this goal, we first build a vector product of the mean-field model of the electromotive force (Eq. 3) with the mean magnetic field $B_0$ and eliminate the term with the coefficient $\alpha$. We obtain an estimator for the coefficient $\beta$ after some vector calculus:

$$\beta_{\text{obs}} = -\frac{1}{H^2} \frac{\nabla \times E_{\text{em}}}{(B_0 \times E_{\text{em}})}, \quad (4)$$

The inputs to the calculation of the coefficients $\alpha$ and $\beta$ are the electromotive force $E_{\text{em}}$, the mean magnetic field $B_0$, and the curl of the mean magnetic field $\nabla \times B_0$. When the measurement is performed by a single-point sensor (or spacecraft) in a supersonic or super-Alfvénic flow, one may limit the gradient direction to the flow direction and introduce an assumption that the spatial derivative is mostly a time derivative with advection by the mean flow $U_0$,

$$\nabla \approx -e_U \frac{\partial}{U_0} t = - \frac{U_0}{U_0^2} \frac{\partial}{\partial t}, \quad (7)$$

where $U_0 = |U_0|$ is the absolute value of the flow speed and $e_U = U_0/U_0$ the unit vector in the direction of the mean flow. Note that Eq. (7) assumes that the spatial gradient is parallel to the mean flow, which reduces the problem to one spatial dimension, and that the spatial derivative is estimated by the time derivative using Eq. (7), which is equivalent to the stationarity of the mean field, $d/dt = \partial/\partial t + U_0 \cdot \nabla = 0$. In this paper, the time derivative is evaluated using the first-order-accuracy backward difference due to the irregularly sampled time series data,

$$\frac{\partial B_0}{\partial t} \bigg|_t \approx \frac{B_0(t) - B_0(t - \Delta t)}{\Delta t}, \quad (8)$$

where $\Delta t$ is the time resolution in the data. Thus, the $x$ component of the curl of the mean magnetic field is evaluated as

$$\nabla \times B_0 \bigg|_x \approx \frac{1}{U_0^2} \left[ -U_0 y \frac{\partial B_{0z}}{\partial t} + U_0 z \frac{\partial B_{0y}}{\partial t} \right] \quad (9)$$

The $y$ and $z$ components of the curl of the mean magnetic field are obtained by circulating $\{x, y, z\}$ into $\{y, z, x\}$ and $\{z, x, y\}$ in Eqs. (9) or (10), respectively.

3 Magnetic cloud in interplanetary space

The estimator for the electromotive force and that for the transport coefficients ($\alpha$ and $\beta$) are tested against a magnetic...
cloud event in interplanetary space using the magnetic field and plasma (ion) data from the Helios 2 spacecraft (Porsche, 1977). The magnetic field data are obtained by the fluxgate magnetometer (also referred to as the saturation core magnetometer or the Forsterosonde magnetometer) (Musmann et al., 1975) and the plasma data by the electrostatic analyzer (Schwenn et al., 1975; Rosenbauer et al., 1977). Merged data between the magnetic field and the plasma measurements are used. The sampling rate varies from 40 s to multitudes of 40 s, and there are data gaps as well. Figure 1 displays the magnetic field magnitude, the proton bulk speed, the proton number density, and the electromotive force magnitude as time series plots from 17 to 20 April 1978. The dots in black represent the original measurements, and the solid lines in gray represent the smoothed data. The Helios 2 spacecraft is in an inbound orbit in the inner heliosphere and moves from a heliocentric distance of 0.41 AU (astronomical unit) on 17 April 1978 to a distance of 0.37 AU on 20 April 1978. A magnetic cloud passes by the spacecraft around 18:00–20:00 UT on 18 April. The magnetic field magnitude increases from about 40 nT to about 70 nT, the ion bulk speed from about 500 to 800 km s\(^{-1}\), and the ion number density from about 30 to about 300 cm\(^{-3}\).

The electromotive force \(E_{em}\) is evaluated by constructing the mean field and the fluctuation field from the time series data. The detailed procedure is as follows.

1. **Mean field determination.** The mean field is determined by assuming the ergodic hypothesis and regarding the mean values in the time domain as statistically representative of ensemble average. We use a fixed time window of 3 h (90 min before the window center and after the center). The averaging period is determined as a compromise such that the fluctuation fields have zero mean values under the conditions of the shortest periods and a sufficient number of data points within the time window. In this work, the smoothing is computed typically over 130 to 270 data points, depending on data availability (due to changes in the sampling rate) within each fixed time window of 3 h. The mean fields for the magnetic field, the flow velocity (for the protons), and the density (again for the protons) at the \(m\)th time record \(t_m\) are determined by the box-car averaging method using the number of data points \(N\) within the time window, e.g., \(B_0(t_m) = \frac{1}{N} \sum_{n=0}^{N-1} B(t_{m+n-N/2})\), for the magnetic field. Other possibilities of the mean field determination are discussed in the last section of the paper.

2. **Fluctuation field determination.** The fluctuation fields are obtained by subtracting the mean field from the measured field, e.g., \(\delta B(t_m) = B(t_m) - B_0(t_m)\), for the magnetic field.

3. **Electromotive force.** The electromotive force is determined in the time domain by building a cross product between the fluctuation flow velocity and the fluctuation magnetic field and then averaging over the same time window as that for the mean fields (again, using the box-car averaging). \(E_{em}(t_m) = \frac{1}{N} \sum_{n=0}^{N-1} \delta U(t_{m+n-N/2}) \times \delta B(t_{m+n-N/2})\).

The electromotive force increases from about \(10^2\) mV km\(^{-1}\) before the magnetic cloud event to above \(10^3\) mV km\(^{-1}\) during the magnetic cloud event. For reference, the order of the electromotive force is 1 mV km\(^{-1}\) for a flow velocity fluctuation of 1 km s\(^{-1}\) and a magnetic field fluctuation of 1 nT.

The transport coefficients \(\alpha\) and \(\beta\) are evaluated using Eqs. (6) and (4), respectively, and their magnitudes are displayed as a function of time in the bottom two panels in Fig. 1. The order of the coefficient \(\alpha\) is 1 km s\(^{-1}\) for an electromotive force of 1 mV km\(^{-1}\) and a mean magnetic field of 1 nT, and that of \(\beta\) is 1 km\(^2\) s\(^{-1}\) for an electromotive force of 1 mV km\(^{-1}\) and a curl of the magnetic field of 1 nT km\(^{-1}\).

The coefficient \(\alpha\) fluctuates around 1 km s\(^{-1}\) overall with an excursion to about \(10^1\) to \(10^2\) km s\(^{-1}\) during the magnetic cloud event. If we use an estimate of 50 nT for the mean magnetic field and \(10^3\) mV km\(^{-1}\) for the electromotive force, we obtain the coefficient \(\alpha\) at about 20 km s\(^{-1}\), which roughly agrees with the peak value of \(\alpha\).

In contrast, the coefficient \(\beta\) has by far larger values throughout the observed time interval and varies by about 4 orders of magnitude between \(10^{10}\) km\(^2\) s\(^{-1}\) at the beginning of 17 April 1978 and a peak of nearly \(10^{14}\) km\(^2\) s\(^{-1}\) around 19:00 UT on 19 April 1978 at the time of magnetic cloud passing. It is interesting to observe that the enhancement of the coefficient \(\beta\) is found not only during the magnetic cloud event but also during other periods, for example, around 02:00–03:00 UT on 18 April 1978 when the magnetic field magnitude is at a local minimum. Another interesting feature is that the variation sense of the coefficient \(\beta\) changes from an anticorrelation sense to that of the coefficient \(\alpha\) (e.g., 00:00–06:00 UT on 17 April 1978) into a positive correlation sense (period on 18 April 1978) and then back into the anticorrelation sense (06:00–18:00 UT on 19 April 1978). Since the coefficient \(\beta\) is an index of turbulent magnetic diffusion, the variation profile of \(\beta\) indicates that turbulence occurs in an inhomogeneous way in the solar wind; some variations are accompanied by the magnetic cloud, others not. A naive estimate for the reason of the large values of the coefficient \(\beta\) is as follows. We use values of \(10^5\) mV km\(^{-1}\) for the electromotive force and \(10^{-5}\) nT km\(^{-1}\) (which is \(10^{-17}\) V s m\(^{-3}\)) for the curl of the mean magnetic field and obtain a value of the coefficient \(\beta\) about \(10^7\) km\(^2\) s\(^{-1}\), which is the lower limit of the measured profile for the coefficient \(\beta\). Here we used a flow speed of 500 km s\(^{-1}\) and a varying timescale of 1000 s for the advection and a magnetic field of 10 nT.

A close inspection shows that the curl of the mean magnetic field is even much smaller than the estimate above and is about \(10^{-10}\) nT km\(^{-1}\), which gives the coefficient \(\beta\) of the order of \(10^{12}\) km\(^2\) s\(^{-1}\). There are various contributions that
suppress the values of the curl of the mean field. The assumption of the one-dimensional advected structure is motivated by the use of single spacecraft data. On the one hand, the alignment of the mean magnetic field with its curl implies the use of a force-free field or a field-aligned current configuration, realized in various space plasma environments. The alignment may have a more fundamental nature. On the other hand, the constraint to the one-dimensional advected structure in the data analysis can be tested against multi-point measurements, from Cluster (Escoubet et al., 2001) or Magnetospheric Multiscale (MMS) (Burch et al., 2016). For example, it would be interesting to compare two different scenarios upon the direction of the spatial gradient using the multi-point data: first, if the gradient of the large-scale fields is mostly aligned with the mean flow direction and, second, if the gradient is perpendicular to the mean magnetic field direction as is the case for solar wind turbulence on ion kinetic scales (at about 400 km) (Perschke et al., 2014). Perhaps the alignment may be scale-dependent from the fluid picture of plasma (on the length scales of the order of 10,000 km) down to ion-kinetic scale (of the order of 400 km), and the small-scale field-aligned currents may be an important component to the turbulent solar wind. Also, continuous measurements at a higher sampling rate are available with the Cluster and MMS missions in comparison to the plasma measurements by the Helios spacecraft limited to 40 s.

4 Outlook

The electromotive force is a second-order quantity such as the energy densities (magnetic energy and kinetic energy) and the helicity densities (magnetic helicity, kinetic helicity, and cross helicity) of the fluctuating fields, but its analysis using the in situ spacecraft data in space plasma has largely been overlooked in earlier studies. Although assumptions have to be incorporated, such as the use of one-dimensional advected structure in the time series data, it is possible to observationally evaluate the electromotive force and determine the transport coefficients using the mean field model for the dynamo theory. Studies on the transport coefficients in the turbulence and dynamo theories can be performed not only by the analytic or numerical methods but also by the observational method. The advantage of the presented method is that even data with different sampling rates can be used to the studies of the electromagnetic force and the transport coefficients at the cost of first-order accuracy approximation in the gradient computation.

Although the proposed method is rather a simple or a naive one, the analysis shows an enhancement of the electromotive force at the magnetic cloud event by 1 or 2 orders of magnitude. This result indicates a scenario that the enhanced or strong magnetic fields in the heliosphere are not merely generated in the Sun or in the solar atmosphere and stream into the heliosphere, but they can be actively amplified in the heliosphere by the flow shear or twist. We believe, however, that the dynamo action is unlikely to occur in the heliosphere. The enhanced electromotive force or the $\alpha$ effect was associated with the magnetic cloud event for a short time. The electromotive force is largest during the magnetic cloud, but the transport coefficients appear to be large also at other times, notably around the minimum magnetic field. For the magnetic field to grow by a dynamo mechanism, the transport coefficients must stay at larger values for a much longer time period and must also operate on different components (toroidal and poloidal components) of the magnetic field.

There are various ways to improve the method presented here. First, from an accuracy point of view, the regularly sampled data are preferred because the second-order central difference method can be applied to the calculation of the derivatives. Second, for a further evaluation of the mean-field dynamo theory, one may test the relations on the transport coefficients $\alpha$ and $\beta$ for a magnetic cloud event observed by Helios 2 spacecraft in the inner heliosphere from 17 to 20 April 1978.

Figure 1. Time series plots of magnetic field magnitude $B$, ion bulk speed $U$, ion density $n$, estimated electromotive force $E_{\text{em}}$ (in the same units as those of the electric field), coefficients $\alpha$ and $\beta$ for a magnetic cloud event observed by Helios 2 spacecraft in the inner heliosphere from 17 to 20 April 1978.
helpful to relax the assumptions of time stationarity and the alignment of the spatial gradient with the mean flow direction.


Competing interests. The authors declare that they have no conflict of interest.

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