On the role of ion-scale whistler waves in space and astrophysical plasma turbulence

Horia Comișel1,2, Yasuhiro Nariyuki3, Yasuhito Narita4,5, and Uwe Motschmann1,6

1 Institut für Theoretische Physik, Technische Universität Braunschweig, Mendelssohnstr. 3, 38106 Braunschweig, Germany
2 Institute of Space Science, Atomiștilor 409, P.O. Box MG-23, 077125 Bucharest, Romania
3 Faculty of Human Development, University of Toyama, 3190 Gofuku, Toyama, 930-8555, Japan
4 Space Research Institute, Austrian Academy of Sciences, Schmiedlstr. 6, 8042 Graz, Austria
5 Institut für Geophysik und extraterrestrische Physik, Technische Universität Braunschweig, Mendelssohnstr. 3, 38106 Braunschweig, Germany
6 Deutsches Zentrum für Luft- und Raumfahrt, Institut für Planetenforschung, Rutherfordstr. 2, 12489 Berlin, Germany

Correspondence to: Horia Comișel (h.comisel@tu-braunschweig.de)

Received: 10 May 2016 – Revised: 28 September 2016 – Accepted: 25 October 2016 – Published: 9 November 2016

Abstract. Competition of linear mode waves is studied numerically to understand the energy cascade mechanism in plasma turbulence on ion-kinetic scales. Hybrid plasma simulations are performed in a 3-D simulation box by pumping large-scale Alfvén waves on the fluid scale. The result is compared with that from our earlier 2-D simulations. We find that the whistler mode is persistently present both in the 2-D and 3-D simulations irrespective of the initial setup, e.g., the amplitude of the initial pumping waves, while all the other modes are excited and damped such that the energy is efficiently transported to thermal energy over non-whistler mode. The simulation results suggest that the whistler mode could transfer the fluctuation energy smoothly from the fluid scale down to the electron-kinetic scale, and justifies the notion of whistler turbulence.

Keywords. Space plasma physics (turbulence)

1 Introduction

Turbulence in space plasmas is fundamentally different from that in ordinary, neutral fluids in that linear mode waves or electromagnetic waves can potentially be a carrier of the fluctuation energy in the spectral domain toward higher wavenumbers. One may speak of weak turbulence if a perturbative approach is valid such that the linear modes play an important role in the energy cascade, and strong turbulence if eddies play an important role in the energy cascade. Another difference of space plasma turbulence from ordinary fluid turbulence is that the inertial range or the energy cascade mechanism can be subdivided into magnetohydrodynamic (MHD) or fluid scale, ion-kinetic scale at wavelengths around the ion inertial length or gyro-radius, and electron-kinetic scale at wavelengths around the electron inertial length or gyro-radius. For example, the transition from the MHD scale to the ion-kinetic scale is about 100 to 1000 km in the solar wind, and that from the ion to electron-kinetic scale is of the order of 10 km.

Naively speaking, there are four fluctuation types that may exist as a linear-mode wave in the ion-kinetic range for a Maxwellian velocity distribution function: whistler, ion Bernstein, kinetic Alfvén, and kinetic slow modes. Figure 1 shows the dispersion relations of these modes at a propagation angle of $\theta_B = 85^\circ$ in a low-beta plasma ($\beta = 0.05$) using the numerical algorithm developed by Gary (1993). These four modes have the following relevance or applications in space plasmas.

- Whistler mode exists not only as a right-hand mode in the parallel direction (to the mean magnetic field) in the cold plasma treatment, but also in the quasi-perpendicular limit. The whistler mode may be regarded as a kinetic extension of the MHD fast mode
of the damping rate magnitude less than the wave frequency. Dashed
Alfvén mode (KA), and kinetic slow mode (KS) under the condition
976 H. Comi¸ sel et al.: Whistler waves
85
85◦

Kinetic Alfvén mode is a kinetic extension of the MHD
–
slow mode (Zhao et al., 2014; Narita and Marsch, 2015)
and its existence is suggested by a pressure-balance
structure in the solar wind (Yao et al., 2013). Kinetic
slow mode is obtained as a quasi-perpendicular limit
of the ion acoustic waves and becomes less damped at
highly oblique angles at around 85◦ and larger. Both
kinetic extensions of the Alfvén and slow modes have
the lowest frequencies (and therefore represent nearly
standing structures) and stay well below the ion gyro-
frequency.

We address the importance of the wave dispersion rela-
onships in plasma turbulence as they can be the primary channel
of the energy cascade mechanism in the inertial range. In this
paper, we report that the whistler mode is the most persistent
mode that survives both in 2-D and 3-D magnetized plas-
as as fluctuations evolve into turbulence. Namely, whistler
turbulence is the most appropriate picture to describe plasma
turbulence on the ion-kinetic scale. We present a study on the
competition of linear wave modes toward turbulence on the
ion-kinetic scales. Ion-scale turbulence represents a transition
from MHD turbulence to electron-scale turbulence, and
may exhibit both wave–wave interactions and wave–particle
interactions. The ion-scale spectral domain may therefore be
regarded as the dispersive–dissipative range.

2 Persistence of whistler mode
We use the method of hybrid plasma simulations in a 3-D
setup for the following reasons. MHD or Hall-MHD sim-
ulations cannot properly resolve wave–particle interactions
for ions. Also, particle-in-cell simulations are numerically
too demanding to perform 3-D turbulence simulations with
a high mass ratio from electrons to ions. Hybrid simulations
treat ions as charged particles (strictly speaking, superparti-
cles using the particle-in-cell algorithm) and electrons as a
massless, finite-pressure fluid as a charge-neutralizing back-
ground.

Our numerical studies follow those presented in Ver-
scharen et al. (2012) and Comišel et al. (2013, 2015) for a
2-D setup. Using the hybrid plasma simulation code AIKEF
(Müller et al., 2011), we solve a set of equations of motion
for ions (treated as superparticles in the particle-in-cell algo-
algorithm) and the Maxwell equations in a self-consistent way.
The equations are time-integrated for 3-D vectorial compo-
nents (e.g., for particle velocity, electric field, and magnetic
field) using a finite-element method in the coordinate space.
Electrons are treated as a finite-pressure massless fluid and
serve as a charge-neutralizing background.

The simulation box size used in the 2-D run has $L_{\perp} = 250 d_i$ and $L_{\parallel} = 250 d_i$ in the perpendicular and parallel direc-

![Figure 1. Dispersion relations calculated for a propagation angle of 85° for low-beta plasma ($\beta = 0.05$), showing whistler modes (WH), ion Bernstein fundamental mode (IB1) and harmonics (IB2), kinetic Alfvén mode (KA), and kinetic slow mode (KS) under the condition of the damping rate magnitude less than the wave frequency. Dashed line shows ion cyclotron mode (IC) for a propagation angle of 75°.](image-url)
Figure 2. Wavenumber spectrum of the magnetic field fluctuations for the 3-D plasma simulation during the time evolution into turbulence.

The wavenumber spectra of the magnetic field fluctuations for the 3-D plasma simulation during the time evolution into turbulence are shown in Fig. 2. The spectra are presented at three different times: $t = 100 \Omega_p^{-1}$, $t = 200 \Omega_p^{-1}$, and $t = 300 \Omega_p^{-1}$. The spectra are shown for three different perpendicular directions: $k_{\perp 1}$, $k_{\perp 2}$, and $k_{\parallel}$. The wave amplitudes are derived with the Kolmogorov power-law scaling (i.e., with the spectral index $-5/3$) up to a cutoff at 20% of the inertial length wavenumber, $k d_i \lesssim 0.2$; see e.g., Verscharen et al. (2012). A number of 18 wave modes are set for the 3-D setup without imposing any power law scaling. Their corresponding wavenumbers are $k_{\perp 1} = 0, \pm 2\pi/L_{\perp 1}$, $k_{\perp 2} = 0, \pm 2\pi/L_{\perp 2}$, and $k_{\parallel} = \pm 2\pi/L_{\parallel}$. The initial wave phases are chosen as random. The wave frequencies are derived from the MHD Alfvén waves, $\omega = \pm k V_A$. No additional pump waves are added during the simulation run, nor is the value of beta reset. The equation of motion and the Maxwell equations are time-integrated in an alternate fashion. The time step of the fast Fourier transformation applied in the time domain is $1\Omega_p^{-1}$ gyro-period while the time range is about $100\Omega_p^{-1}$ gyro-periods.

The initial spectrum is established such that the magnetic field fluctuations in the coordinate space is 1% of the mean magnetic field and this value is implicitly assumed in our simulation results unless a higher amplitude of 10% is specified. Verscharen et al. (2012) discussed the regime of turbulent cascade and the role of the fluctuation amplitude of the initial pumping waves in their study at ion beta 0.05. The authors noticed that the turbulent cascade is more pronounced at larger pumping-wave amplitudes (10% of the mean magnetic field). The magnetic energy spectrum in the wavenumber–frequency domain shows significant amount of energy only along the dispersion relation for the whistler mode.

The spectral decay in the wavenumber domain for the 3-D simulation run is given in Fig. 2 at three different times: $t \Omega_p = 100$, $t \Omega_p = 200$, and $t \Omega_p = 300$. While the spectrum is rather isotropic in the perpendicular wavenumber domain (Fig. 2, upper row), the spectral decay is flatter in the perpendicular direction and steeper in the parallel direction (Fig. 2, bottom row).

We are aware that at this small 1% initial amplitude, it is difficult to clearly distinguish the flow of the energy from MHD scales to the kinetic scales. Figure 3a shows the 1-D
wavenumber spectrum of the magnetic field fluctuations obtained at time $T = 500 \Omega_p^{-1}$ (solid black line). The gray line indicates the spectral curve from a simulation run without any initial pump. The black dashed line represents the slope of the Kolmogorov spectrum ($-5/3$). At larger wavenumbers, the overlap of the spectra for the initial MHD pumping waves and the thermal noise can have a physical interpretation like a mixture between fluctuations originating in the Alfvén wave excitation and the thermal noise manifested by the solar wind plasma. As an additional comment, in order to minimize the consequences of the numerical noise, we carried out highly demanding computational runs by using more than $10^{10}$ superparticles in the simulation box.

In the 2-D simulation, the energy spectrum develops by showing various dispersion relations. At a time of 600 ion gyro-periods, whistler mode, ion Bernstein modes, and ion cyclotron mode are clearly visible in the wavenumber–frequency spectrum (as a slice of along the $k_\perp$ axis) in Fig. 4, obtained by using data from Comisel et al. (2013). This earlier study has found that the frequencies follow first for the linear modes, and then deviate or become broadened from the linear modes.

In the 3-D simulation, in contrast to the 2-D case, the energy spectrum develops primarily by showing the whistler mode branch. Strong ion Bernstein modes no longer appear. The low-frequency modes such as the ion cyclotron mode and the kinetic slow mode cannot be clearly identified because of the broad frequency distribution in a wider range of the wavenumbers. Figure 5a displays a slice of the magnetic energy spectrum along the $k_\perp$ axis. The fluctuation energy is axi-symmetrically distributed around the mean magnetic field direction. Only the whistler mode can be mainly identified in the sliced spectrum along the $k_{\perp 2}$ axis during the simulation run.

### 3 Discussion and outlook

Why does only the whistler mode survive during the plasma evolution into turbulence in the 3-D coordinate space and the other modes do not? In fact, one may expect that the ions should be heated by the dissipation of the left-hand modes. In the 3-D simulation, two different scenarios could explain the
missing ion Bernstein (IB) modes: (1) the IB modes are excited but the resonance with the ions is so efficient that the energy of the IB modes goes immediately into thermal energy and (2) the IB modes are not excited in the 3-D setup and the ion heating is not significantly occurring. Consequently, we first search for evidence of increasing ion temperature. The total temperature increase in the simulation box at the latest time of the 3-D run ($t \sim 700 \Omega_{pi}^{-1}$) is about 8% of the initial temperature. This temperature increase along the parallel direction cannot account for the particle heating provided by the dissipation of the IB modes. For example, in the 2-D scenario, the perpendicular temperature increase is about 70% of the initial temperature; see e.g., Verscharen et al. (2012).

For a better clarification of the above proposed scenarios, the evidence for wave damping is investigated by checking the occurrence of the left-hand mode. We apply a method of decomposition of the right-hand (R) and left-hand (L) modes based on the Stokes parameters $\{I, Q, U, V\}$ from the magnetic field. The decomposition into R and L modes is meaningful for field-aligned and oblique wave propagation. The procedure is described in Appendix A. Each component of the fluctuating field is completed from real into complex values by shifting a phase of 90° using the Hilbert transformation. The phase information in each component is used to determine the rotation sense of fluctuation. The method is calibrated by exciting the ion cyclotron instability. A 1-D hybrid simulation is performed starting with a high initial temperature anisotropy; see e.g., Gary and Saito (2003). The left-hand mode is assigned to the strongest branch resulting in the decomposition method. Figure 6a shows the energy of the right-hand mode $E_R$ (solid black line) and the left-hand mode $E_L$ (solid gray line) with respect to the elapsed time in the simulation. Both curves evolve smoothly at the same level until time $t \Omega_{pi} \sim 150$, then they quickly begin growing in magnitude. At about time $t \Omega_{pi} \sim 300$, the left-hand mode becomes clearly stronger than the right-hand mode. The difference between the two modes initially increases but at later times (not shown in Fig. 6.), the left-hand modes slowly achieve a decaying phase. We interpret this result as a coupling between the MHD Alfvén waves and the fast magnetosonic waves at time $t \Omega_{pi} \sim 150$. The energy is pumped from MHD scale in the system and both the R and L modes rapidly start to grow until time $t \Omega_{pi} \sim 1000$. The oscillations seen at the latest time in both R and L modes are provided by the vibration of the mean magnetic field induced by the MHD Alfvénic wave. The oscillations have amplitudes at 1° from the mean magnetic field direction while the mean magnetic field is now tilted at an angle of 3.5°. In this regime, the wavenumber–frequency spectrum (not shown here) is closer to that one obtained by Verscharen et al. (2012) by using 10% amplitude of the pumping waves; the persistent linear wave modes are the whistlers. The other left-hand modes have been decayed.

The right- and left-hand modes obtained by decomposing the magnetic field fluctuations from the 3-D setup are also plotted in Fig. 6a by using the same color convention (black line for R mode, gray line for L mode). $E_R$ and $E_L$ oscillate due to the initial MHD excitation and smoothly increase at later times due to a weak inclination of the mean magnetic field. Their profiles do not encounter the exaltation of the 2-D analogous modes. The circular polarization of the waves is weaker than in the 2-D setup and the waves are much more linearly polarized.

An additional 3-D run is carried out for a further investigation on the role played by the pumping Alfvén waves for the persistency of the whistler modes. In order to attain a clear energy cascade from the MHD scale to the ion-kinetic scale, an asymmetric box is set with a diminished length on the direction parallel with respect to the mean magnetic field ($L_1 = 64d_i$). The current number of superparticles in the computational cell is doubled, from 200 superparticles to 400 superparticles, the ion beta parameter is decreased from $\beta_i = 0.1$ to $\beta_i = 0.05$, and the amplitude of the initial pumping waves is raised at a value of 10% from the mean magnetic field. Figure 3b shows, by using the incremental grayscale color nuances, the 1-D reduced power spectra along the perpendicular wavenumber axis at times 0, 100, 200, 300, 400, and 500 ion gyro-periods. At the initial time, the power spectrum is given at larger scales by the solid, light gray line. At a time of 100 gyro-periods, the energy...
is transported at lower scales by quickly decreasing until a minimum value is reached at the wavenumber \( k V_A / \Omega_p \approx 1 \).

At later times (\( t > 100 \Omega_p^{-1} \)), the fluctuation level is higher than the noise level while the spectral slope is attending the Kolmogorov value of \(-5/3\). The turbulent fluctuations reach a quasi-stationary state (in terms of the energy spectra) by 500 ion gyro-periods.

The wavenumber–frequency spectrum is shown at a time of 500 gyro-periods in Fig. 5b. The whistler mode is the wave mode solely excited at the perpendicular direction with respect to the mean magnetic field. The other weak modes observed in Fig. 5a (in particular the second harmonic of Bernstein mode) are suppressed. The result of the decomposition method is shown in Fig. 6c. Left- and right-hand modes are clearly excited in the sense that their profiles are uncorrelated during the time evolution of plasma turbulence. Figure 6b shows, for comparison purposes, the result obtained for the 2-D setup using equivalent driving amplitudes. At the time when the energy is about to cascade at smaller scales (\( t > 100 \Omega_p^{-1} \)), the strength of both left- and right-hand modes starts to grow. In contrast with the former 3-D setup, Fig. 7 shows a temperature increasing in the perpendicular direction of about 15% from the initial value. The coupling between oblique whistler modes and oblique Bernstein modes has been proved by Markovskii et al. (2010) to manifest as a proper mechanism for heating the protons in a 2-D hybrid simulation by using initial fast magnetosonic waves. A similar process, seen as an indirect heating by means of the whistler waves, may occur in our second 3-D setup, and then the first assumed scenario at the beginning of this section is available. Nevertheless, whistler mode is persistent irrespective of dimensionality or initial setup because all other modes are excited and immediately damp such that the energy is efficiently transported to thermal energy over non-whistler mode.

We summarize the 3-D simulation results as follows. By applying lower driving amplitude waves, the IB modes or other modes are weakly excited – the opposite of the 2-D result. The decomposition method shows weak evidence of circular polarized waves at parallel and oblique propagation angles with respect to the mean magnetic field. When higher driving amplitude waves are used, the suppression of the linear modes except for the whistler modes is observed, as shown in the 2-D simulation of Verscharen et al. (2012). Actually, the detailed process of the suppression and the appearance of the outstanding whistler mode (fast mode) are still unclear even in the present study using the 3-D simulation results. But it is worth noting that the outstanding whistler waves, which is similar to those in the 2-D simulation of Verscharen et al. (2012), also appears in the 3-D simulation, even though the only smallest wavenumber is given as the initial condition of the 3-D setup. This suggests that the appearance of the whistler mode itself is relatively robust for the difference of the initial shape of wavenumber spectra. In contrast, the difference between the 2-D and 3-D setup in the case of lower driving amplitude wave possibly occurs due to the difference of the initial conditions. From this point of view, the difference of temperature in the case of higher driving amplitude waves can also come from the difference of the initial conditions. To clarify these points and scenarios, more parameter studies are necessary.

As an alternative scenario, the azimuthal degree of freedom blocks the development of the electrostatic component such that the IB modes (which have a larger electrostatic component) are suppressed and the whistler mode (which is an electromagnetic mode) can evolve. Ganguli et al. (2010) discussed on the 3-D character of the electromagnetic whistler turbulence in the intermediate frequency range above the proton gyro-frequency and below the electron gyro-frequency at low beta plasmas. The authors’ opinion is that the wave–particle scattering can convert electrostatic waves with low group velocity into electromagnetic waves with large group velocity. This process can convert energy away from the region with the result of a wave energy loss without significant local particle heating.

The lessons from the 3-D hybrid simulation of plasma turbulence can be resumed as follows. The whistler mode is persistent in hybrid simulation, and could fill the gap of the energy cascade process between MHD turbulence and electron-scale turbulence. The existence of whistlers as the only persistent mode justifies the previously proposed scenario that whistlers are the pump or a major energy carrier for electron-kinetic scale plasma turbulence (Saito et al., 2008, 2010; Gary et al., 2012; Chang et al., 2013). In fact, Narita et al. (2016b) has recently reported new findings of the whistler turbulence in the solar wind at electron scales for oblique propagation angles. The other candidate wave modes with quasi-perpendicular wavevectors such as kinetic Alfvén mode, kinetic slow mode, and IB modes are either not excited by wave–wave couplings or strongly damped. The IB mode exists only in a 2-D domain.

We note possible applications of our findings. The persistency of whistler mode motivates us to construct a phenomenological model for whistler turbulence as formulated...

Figure 7. Time evolution of the perpendicular (black) and parallel (gray) temperature of the protons for the 3-D setup using higher driving amplitude waves.
earlier by Narita and Gary (2010) and Saito et al. (2010). For example, it would be interesting to extend the critical balance hypothesis to the ion-kinetic scale by balancing the eddy turnover time and the whistler wave scatter time. On the other hand, search for the ion-scale whistler and the IB modes is a suitable task to understand the turbulent heating process in the inner heliosphere in the upcoming Solar Orbiter and Solar Probe Plus missions.

4 Data availability

Data from our 2-D and 3-D hybrid simulations supporting the results presented in this paper are stored at the Institut für Theoretische Physik – Technische Universität Braunschweig. Data can be obtained by writing to the following email addresses: h.comisel@tu-braunschweig.de or comisel@spacescience.ro.
Appendix A: Stokes parameters

The Stokes parameters (Stokes, 1852) uniquely decompose the fluctuation energy for the transverse waves (with respect to the specified axis) into different bases such as linear or circular polarizations. Here we choose the mean magnetic field direction as the specified axis, and compute the Stokes parameters for the magnetic field fluctuations perpendicular to the mean field. More detailed explanations of the Stokes parameters are found in a review article by Berry et al. (1977) and a textbook by Born and Wolf (1980). In our present work, the Stokes parameters are determined for the magnetic field fluctuations perpendicular to the mean field are analyzed. The four Stokes parameters, $I$, $Q$, $U$, and $V$, are expressed using linear polarization basis $\{b_x, b_y\}$ or circular polarization basis $\{b_R, b_L\}$ as follows:

\[
I = \langle |b_x|^2 \rangle + \langle |b_y|^2 \rangle = \langle |b_R|^2 \rangle + \langle |b_L|^2 \rangle \tag{A1}
\]

\[
Q = \langle |b_x|^2 \rangle - \langle |b_y|^2 \rangle = 2\text{Re} \left( \langle b_R b_L^* \rangle \right) \tag{A2}
\]

\[
U = 2\text{Re} \left( \langle b_x b_y^* \rangle \right) = -2\text{Im} \left( \langle b_R b_L^* \rangle \right) \tag{A3}
\]

\[
V = -2\text{Im} \left( \langle b_x b_y^* \rangle \right) = \langle |b_R|^2 \rangle - \langle |b_L|^2 \rangle. \tag{A4}
\]

The Stokes parameter $I$ represents the total amount of fluctuation energy averaged over a suitable ensemble. We average over the spatial domain in the present work. The Stokes parameter $Q$ is a measure of the energy difference between the $x$ and $y$ axes, $U$ the energy difference between linear polarization fields along the axes rotated from $x$ and $y$ axes by $45^\circ$, and $V$ the energy difference between the right- and left-hand circular polarized fields. The fluctuation fields $b_x$ and $b_y$ are given as complex values, and are obtained by combining the measured real-value fields $B_x$ and $B_y$ with their Hilbert transforms, $\hat{B}_x$ and $\hat{B}_y$:

\[
b_x = B_x + i\hat{B}_x \tag{A5}
\]

\[
b_y = B_y + i\hat{B}_y, \tag{A6}
\]

respectively, where $i$ denotes the imaginary unit. The fluctuation energies for the right- and left-hand circular polarizations are obtained from $I$ and $V$ as

\[
E_R = \frac{I + V}{2} \tag{A7}
\]

\[
E_L = \frac{I - V}{2}, \tag{A8}
\]

respectively.
Acknowledgements. This work was financially supported by the Collaborative Research Centre 963: Astrophysical Flow, Instabilities, and Turbulence of the German Science Foundation. The work conducted by H. Comisel in Bucharest is supported by the Romanian National Authority for Scientific Research and Innovation, CNCS – UEFISCEDI, project no. PN-II-RU-TE-2014-4-2420. H. Comisel is grateful for the JSPS Invitation Fellowship for Research in Japan (ID no. S15131) and thankful for the hospitality at the University of Toyama. We acknowledge the North-German Supercomputing Alliance (Norddeutscher Verbund zur Förderung des Hoch- und Höchstleistungsrechnens – HLRN) and Jülich Supercomputing Centre (JSC) JURECA for supporting our direct numerical simulations.

The topical editor, G. Balasis, thanks one anonymous referee for help in evaluating this paper.

References


www.ann-geophys.net/34/975/2016/


