

The correlation between solar and geomagnetic activity – Part 3: An integral response model

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Abstract. An integral response model is proposed to describe the relationship between geomagnetic activity (*aa* index) and solar activity (represented by sunspot number R_z): The *aa* at a given time t is the integral of R_z at past times ($t' \leq t$) multiplied by an exponential decay factor of the time differences ($e^{-(t-t')/\tau}$), where τ is the decay time scale (~ 40 months). The correlation coefficient of *aa* with the reconstructed series based on this model ($r_f = 0.85$) is much higher than that of *aa* with R_z ($r_0 = 0.61$). If this model is applied to each solar cycle, the correlation coefficient will be higher ($r_f = 0.95$). This model can naturally explain some phenomena related to *aa* and R_z , such as (i) the significant increase in the *aa* index (and its baseline) over the twentieth century; (ii) the longer lag times of *aa* to R_z at solar cycle maxima than at minima; and (iii) the variations in the correlations related to solar and Hale cycles. These results demonstrate that *aa* depends not only on the present R_z but also on past values. The profile of *aa* can be better predicted from R_z by this model than by point-point correspondence.

Keywords. Solar physics, astrophysics, and astronomy (General or miscellaneous)

1 Introduction

Studying the relationship between solar and geomagnetic activity is useful for understanding the origin and formation of the latter. The geomagnetic activity index *aa*, calculated from the 3-hourly K indices measured at two near-antipodal midlatitude stations (Mayaud, 1972), has been used for an-

alyzing long-term trends in the global geomagnetic activity and in its correlation with solar activity (Schatten et al., 1978; Feynman, 1982; Legrand and Simon, 1989; Nevanlinna and Kataja, 1993; Schatten and Pesnell, 1993; Russell and Mulligan, 1995; Mursula et al., 2004; Prestes et al., 2006; Cameron and Schüssler, 2007; Du et al., 2009; Lukianova et al., 2009; Du and Wang, 2011). It has an 11-year variation similar to that of the solar activity, as described by the Zurich relative sunspot number (R_z). In the twentieth century, there has been a significant increase in the *aa* index (and its baseline), the reason for which, however, is unknown (Feynman and Crooker, 1978; Clilverd et al., 1998; Demetrescu and Dobrica, 2008; Lukianova et al., 2009).

The solar activity has long been recognized to be at the origin of the geomagnetic activity (Snyder et al., 1963; Russell and McPherron, 1973; Garrett et al., 1974; Feynman, 1980; Legrand and Simon, 1981). The geomagnetic activity is the result of variable current systems formed in the magnetosphere and ionosphere as a consequence of the interaction of the solar wind with the magnetosphere (Gonzalez and Tsurutani, 1987; Gonzalez et al., 1989, 1994; Demetrescu and Dobrica, 2008). The geomagnetic activity has been found to be well correlated with the solar wind speed (v), the southward component (B_z) of the interplanetary magnetic field (IMF) and the product $B_z v^2$ (Snyder et al., 1963; Russell and McPherron, 1973; Garrett et al., 1974; Crooker et al., 1977; Svalgaard, 1977; Feynman, 1980; Tsurutani et al., 1988; Wang and Sheeley, 2009).

Different solar and geomagnetic activities tend to peak at different times relative to the peak of each sunspot cycle. For example, the upper chromospheric activity indices (Bachmann and White, 1994) and the solar flares (Wheatland and Litvinenko, 2001; Temmer et al., 2003) tend to lag behind the sunspot number by one to several months, depending on the



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index, which was interpreted in terms of active regions evolving from the photosphere upward. The solar transient activity (e.g., solar flares) dominates the rising phase (Borello-Filisetti et al., 1992), while recurrent geomagnetic activity is more frequent during the declining phase or at the minimum of solar cycle (Legrand and Simon, 1989; Sargent, 1985; Tsurutani et al., 1995; Venkatesan et al., 1991; Echer et al., 2004; Tsurutani et al., 2006; Richardson and Cane, 2002). During the last years of a cycle, the geomagnetic activity results from recurrent storms, fast solar winds, and coronal holes (Svalgaard, 1977; Legrand and Simon, 1981). The solar magnetic field that permeates the corona strongly modulates these activities (Schwenn, 2006).

A determining role in the formation and dynamics of solar activity is played by magnetic fields. Magnetic fields fill the solar atmosphere – from underlying the solar surface to the outer atmosphere, the solar corona – and have been linked with changes in total cloud cover over the Earth, which may influence global climate (Lockwood et al., 1999). Sunspots represent one of the most obvious manifestations of local magnetic fields on the Sun, the main sites of solar-activity phenomena (Moradi et al., 2010), and have been considered as a measure for the energy supply to the corona (Temmer et al., 2003). The magnetic field and its topology in the corona are closely related to the magnetic field on the solar photosphere (Sakurai, 1981; Yan and Li, 2006), and can be analyzed by potential or non-potential field extrapolation from the photospheric field (Sakurai, 1981; Tu and Marsch, 1995; Yan and Li, 2006). Coronal holes are the origins of high-speed solar wind streams (Parker, 1963; Gosling and Pizzo, 1999; Tsurutani et al., 1995, 2006; Gonzalez et al., 2004). Therefore, it is natural that there are direct or indirect connections between the aa geomagnetic index and the sunspot number R_z .

How does the solar activity affect the geomagnetic activity, linearly or nonlinearly? In what manner? Although it has been well known that the solar activity is the main source of geomagnetic activity, there are no accurate expressions to clearly describe the relationship between them. It has been established that the magnetosphere has a significant linear response to the solar wind drivers. In the declining phase of a solar cycle, however, the dynamics of the magnetosphere exhibit a nonlinear behavior (Johnson and Wing, 2005). The increasing occurrence of high-speed solar wind streams during the declining phase of the cycle, whose reason needs to be explained, has been used to explain the decreasing trend in the correlation between aa and R_z over time (Bame et al., 1976; Borello-Filisetti et al., 1992; Mussino et al., 1994; Tsurutani et al., 1995; Kishcha et al., 1999).

What is the reason for the significant increase in the aa index (and its baseline) over the twentieth century? Why does the aa index tends to lag behind R_z about 2–3 years around a solar cycle maximum (Wang et al., 2000; Echer et al., 2004), while around a cycle minimum the lag time is small, at about 1 year (Wilson, 1990; Wang and Sheeley, 2009)? Why is the

aa index strongly correlated with R_z at the rising phase of a solar cycle, while the correlation at the declining phase is weak and decreases with time (Borello-Filisetti et al., 1992; Mussino et al., 1994; Kishcha et al., 1999; Echer et al., 2004; Du, 2011b)?

Conventionally, the relationship between aa and R_z has been analyzed by point-point correspondence. However, some of the above questions are hardly understood and the correlation between them is unsatisfactory whether considering the time delay of aa to R_z or not. This paper investigates the systematic relationship between aa and R_z by an integral response model of aa to R_z . In this model, the output (aa) depends not only on the present input (R_z) but also on past values. Using this model, the above questions (Du, 2011b) can be naturally explained. Section 2 shows the results for all data since the onset of Cycle 12 followed by explanations for the above questions in Sect. 3. Section 4 shows the results when this model is applied to each cycle of 12 through 23. Our conclusions are discussed and summarized in Sect. 5.

2 Results

The data used in this study are the time series of monthly mean aa geomagnetic index, representing the geomagnetic activity (Mayaud, 1972), of the reliable values since 1868¹ together with the monthly mean sunspot number (R_z)², representing the solar activity. To filter out high frequency variations, the data have been smoothed with a 24-month Gaussian filter. The relative weights are given by

$$W(\Delta t) = \exp[-2\Delta t^2/b^2] - \exp[-2](3 - 2\Delta t^2/b^2), \quad (1)$$

where Δt is the number of months from the center and b ($= 24$ months) is the full width at half maximum (Hathaway et al., 2002). For comparison with the results of individual cycles, we employ the data from the onset of Cycle 12 (August 1878) to the end of Cycle 23 (August 2008), as shown in Fig. 1: $aa - 5$ (solid line, shifted downward by 5 for clarity) and R_z (dashed) to the right of the vertical dash-dotted line. The correlation coefficient between aa and R_z is $r_0 = 0.61$ – about only one-third ($r_0^2 = 37.2\%$) of the variation in aa can be explained by a linear correlation.

2.1 The integral response model of aa to R_z

Generally, the output $O(t)$ of a system will depend not only on the present input $I(t)$, but also on past values. Approximately, $O(t)$ is a weighted sum of the previous values of $I(t')$, with the weights given by a response function $h(t - t')$,

$$O(t) = \int_{-\infty}^t I(t')h(t - t')dt'. \quad (2)$$

¹ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/RELATED-INDICES/AA_INDEX/

²<http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspot-number.html>

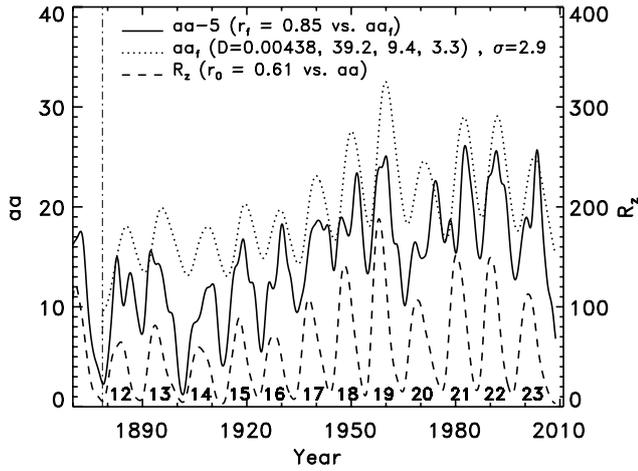


Fig. 1. Monthly mean $aa - 5$ (solid) and R_z (dashed) smoothed with a 24-month Gaussian filter. The dotted line shows the reconstructed aa series (aa_f) from August 1878 through August 2008 (Cycles 12–23) by Model (7). The correlation coefficients of aa with R_z and aa_f are $r_0 = 0.61$ and $r_f = 0.85$, respectively.

In this study, the output is aa and the input is R_z . Now, we determine the explicit expression of Eq. (2). Suppose that at time $t = t'$ there is an input from solar activity, which is approximately represented by $R_z(t')$, and that this input generates an output of geomagnetic activity linearly correlated with $R_z(t')$,

$$aa'(t') = D(R_z(t') + R_0), \quad (3)$$

where D and R_0 are constants, the prime on aa represents the part of aa that is generated by $R_z(t')$.

This activity then undergoes a decay process according to time t (due to variable current systems *etc*),

$$-\Delta aa' \propto aa' \Delta t, \quad \text{or} \quad \frac{\partial aa'}{\partial t} = -\frac{1}{\tau} aa', \quad (4)$$

where τ indicates the decay time scale. Its solution is

$$aa'(t) = e^{-(t-t')/\tau} aa'(t'), \quad (5)$$

where $aa'(t')$ is the integral constant at $t = t'$, which has been already assumed to be a linear function of $R_z(t')$ at the initial time $t = t'$ (Eq. 3). So that,

$$aa'(t) = D(R_z(t') + R_0)e^{-(t-t')/\tau}. \quad (6)$$

This equation means that a solar activity ($R_z(t')$) at time t' will generate a series of geomagnetic activities ($aa'(t)$) in the subsequent times ($t > t'$) according to an exponential decay factor ($e^{-(t-t')/\tau}$). In other words, the geomagnetic activity at time t is generated by all the solar activities before time t , i.e., the summation of Eq. (6),

$$\begin{aligned} aa(t) &= \sum aa'(t) + aa_0 \\ &= D \int_{t'=-\infty}^t [R_z(t') + R_0] e^{-(t-t')/\tau} dt' + aa_0 \\ &= D \sum_{t'=t_0}^t [R_z(t') + R_0] e^{-(t-t')/\tau} + aa_0. \end{aligned} \quad (7)$$

In this expression, the input is the solar activity ($I(t') \propto R_z(t')$), the output is the geomagnetic activity ($O(t) \propto aa(t)$), and the response function $h(t-t') \propto e^{-(t-t')/\tau}$. The summation is taken over from the starting time (t_0) of the series to time t .

The meanings of the parameters are as follows: (i) D is called “Dynamic response factor”, representing the initial generation efficiency of geomagnetic activity $\partial aa_i / \partial R_z$; (ii) τ is called “response time scale” of aa to R_z , representing that a solar activity may generate a series of geomagnetic activities in the subsequent time period of about τ (months); (iii) aa_0 is a constant, representing the geomagnetic activity generated by earlier solar activities (see Sects. 3.1 and 4.2), and (iv) R_0 represents that some weak solar activities (magnetic fields), which should be but have not been seen in the form of sunspots (the sunspots or dark pores are too small to be seen), may also generate geomagnetic activities. Penn and Livingston (2006) pointed out that the magnetic field has a threshold of 1500 Gauss, below which no dark pores formed, which represents a real physical limit for the formation of a dark spot (either a pore or a sunspot) on the solar photosphere. Weak magnetic fields may be insufficient to form sunspots, but can generate geomagnetic activities. The value of R_0 just reflects the effect of these weak fields.

2.2 Four-parameter model

Firstly, we use model (7) to fit the aa series from the onset of Cycle 12 through the end of Cycle 23. The four parameters by a least-squares-fit are

$$\begin{cases} D = 0.00438 \pm 0.00004, \\ \tau = 39.2 \pm 0.4 \text{ (month)}, \\ aa_0 = 9.4 \pm 0.2, \\ R_0 = 3.3 \pm 1.6, \end{cases} \quad (8)$$

where \pm represents the standard deviation.

Using these parameters and Model (7), the aa series can be reconstructed, as shown in Fig. 1 (aa_f , dotted line). It is seen that aa_f well reflects the profile of aa , the time delay of aa to R_z , and the increase in aa or the baseline (the aa minimum of geomagnetic cycle, or simply the aa minimum, aa_{\min}) over the twentieth century. The standard deviation of the reconstruction is $\sigma = 2.9$. The correlation coefficient between aa and the reconstructed series (aa_f) is $r_f = 0.85$, much higher than the original value ($r_0 = 0.61$) between aa and R_z . This means that about two-thirds ($r_f^2 = 72.3\%$) of the variation in aa can be explained by Model (7), much higher than that ($r_0^2 = 37.2\%$) for a linear correlation.

One may argue that, if considering the time delay of aa to R_z , the correlation between aa and R_z will also be improved. To show this, we calculate the (linear) correlation function (r) between R_z and aa of the lag $L = -200, -199, \dots, 200$, as shown in Fig. 2a. It is seen that r varies with a periodicity of about 129 months, which represents the (~ 11 -year) Schwabe cycle in both R_z and aa . The maximum value

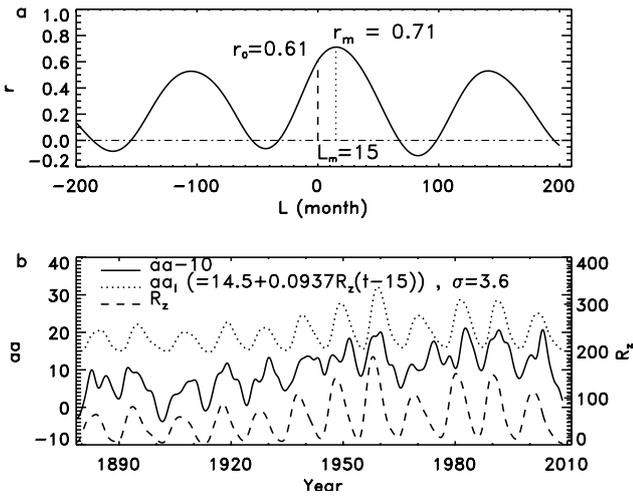


Fig. 2. (a) Correlation function between R_z and aa of the lag $L = -200, -199, \dots, 200$ (months). (b) $aa - 10$ (solid), R_z (dashed), and the reconstructed series aa_1 (dotted) from a linear fit of aa to the R_z fifteen months earlier (9).

is $r_m = 0.71$ at a lag of $L_m = 15$ (months). The best-fit equation of $aa(t)$ against $R_z(t - 15)$ is given by

$$aa(t) = 14.5 \pm 0.2 + (0.0937 \pm 0.0022)R_z(t - 15). \quad (9)$$

The standard deviation of the regression equation is $\sigma = 3.6$. Figure 2b shows $aa - 10$ (solid, shifted downward by 10 for clarity), R_z (dashed), and the reconstructed series aa_1 (dotted) from this linear relationship (Eq. 9). Even if considering the lag time (L_m) of aa to R_z , the correlation coefficient (0.71) between aa and aa_1 is still lower than that ($r_f = 0.85$) from Model (7) in Fig. 1. In addition, although aa_1 can approximately indicate the aa maxima (aa_{max}), while the reconstructed aa minima from aa_1 almost keep a constant level (from 15.0 to 14.8 in the range of [14.8, 16.0] with a rising factor of $14.8/15.0 \sim 1.0$) due to the small values of the solar minima (R_{min}) and thus can not reflect the increasing trend in the baseline (aa_{min}) of aa . Therefore, to describe the relationship between aa and R_z , model (7) is more appropriate than a simple linear function.

2.3 Three-parameter model

When using Model (7) to fit the aa time series in the above section, the R_0 value is small (3.3). Therefore, we neglect R_0 and use the following three-parameter expression to refit the aa series for Cycles 12–23,

$$aa(t) = D \int_{t'=-\infty}^t R_z(t') e^{-(t-t')/\tau} dt' + aa_0 \quad (10)$$

$$= D \sum_{t'=t_0}^t R_z(t') e^{-(t-t')/\tau} + aa_0.$$

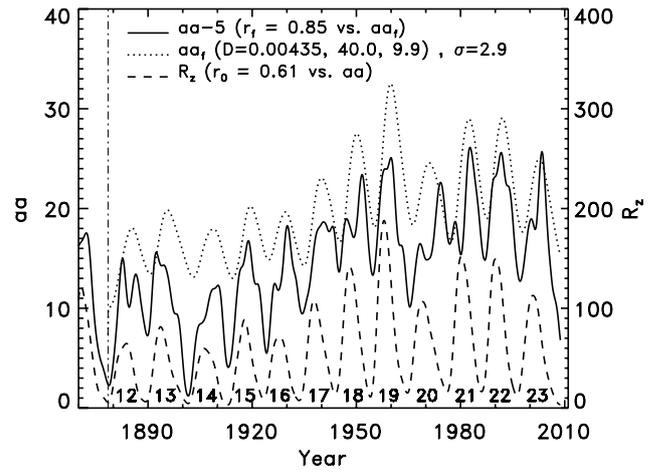


Fig. 3. Similar to Fig. 1 but using three-parameter Model (10). The correlation coefficient of aa with the reconstructed series aa_f is also $r_f = 0.85$.

The reconstructed aa series (aa_f , dotted) is shown in Fig. 3. The three parameters based on this model,

$$\begin{cases} D = 0.00435 \pm 0.00004, \\ \tau = 40.0 \pm 0.4 \text{ (month)}, \\ aa_0 = 9.9 \pm 0.1, \end{cases} \quad (11)$$

have no significant changes compared with those for the four-parameter Model (8). The standard deviation of the reconstruction keeps the same level, $\sigma = 2.9$. As the R_0 value in Eq. (8) is very small (3.3) and the correlation coefficient between aa and the reconstructed series aa_f in this case ($r_f = 0.85$) is equal to that in the case for four-parameter model in Fig. 1, the effect of three-parameter Model (10) is equivalent to that of four-parameter Model (7). Therefore, we will use the three-parameter Model (10) in the following sections.

3 Explanations for some correlations of aa with R_z

It is seen in Fig. 3 that the reconstructed series (aa_f) generally reflects the profile of aa , the time delay of aa to R_z , and the increase in aa over the twentieth century. Therefore, Model (10) well represents the response of geomagnetic to solar activity, and can naturally explain in part the relationship between aa and R_z .

For convenience, Table 1 lists the maximum amplitude of sunspot cycle (R_{max}), the preceding aa minimum (aa_{min}), the aa maximum of geomagnetic cycle (aa_{max}), the lag times of aa to R_z at solar minimum (L_{min}) and at solar maximum (L_{max}), and the corresponding values ($aa_{min,f}$, $aa_{max,f}$, $L_{min,f}$, $L_{max,f}$) from the reconstructed series aa_f in Fig. 3. The last two rows in Table 1 indicate the averages of these values and the correlation coefficients of these values with R_{max} .

Table 1. Geomagnetic minimum (aa_{\min}), maximum aa (aa_{\max}), lag times of aa to R_z at minimum (L_{\min}) and at maximum (L_{\max}) for Cycles $n = 12$ – 23 , and the corresponding values ($aa_{\min,f}$, $aa_{\max,f}$, $L_{\min,f}$, $L_{\max,f}$) from the reconstructed series (10). The last two rows show the averages of these values and the correlation coefficients (r) of these values with the maximum amplitude (R_{\max}) of sunspot cycle.

n	R_{\max}	Observed				Fitted			
		aa_{\min}	aa_{\max}	L_{\min}	L_{\max}	$aa_{\min,f}$	$aa_{\max,f}$	$L_{\min,f}$	$L_{\max,f}$
12	64.7	7.3	20.0	3	−14	7.3	18.0	0	19
13	81.4	12.3	20.6	6	−15	13.4	19.8	16	23
14	59.6	6.2	17.3	0	47	13.1	18.0	18	31
15	88.6	9.1	21.8	3	13	13.0	20.2	19	20
16	71.6	10.5	23.3	11	27	14.8	19.7	18	21
17	108.2	14.4	23.7	11	33	14.3	23.1	17	26
18	141.7	18.6	28.4	13	43	16.7	27.5	16	24
19	188.0	18.3	30.1	9	24	18.1	32.5	15	23
20	106.6	15.1	27.6	9	62	18.4	24.6	19	22
21	151.8	20.4	31.1	48	30	16.7	29.0	15	26
22	149.2	19.8	30.6	12	20	19.0	29.1	19	25
23	112.8	17.7	30.7	11	31	17.7	25.1	19	23
\bar{x}	110.4	14.1	25.4	11	25	15.2	23.9	16	24
r	1.00	0.89	0.86	0.50	0.21	0.73	0.99	0.23	0.08

3.1 The increase in aa over the twentieth century

It is well known that, in the twentieth century, there has been a significant increase in the aa index (and its baseline), the reason for which, however, is unknown (Feynman and Crooker, 1978; Clilverd et al., 1998; Demetrescu and Dobrica, 2008; Lukianova et al., 2009).

Figure 4a shows the values of aa_{\min} (solid), $aa_{\min,f}$ (dotted) and R_{\max} (dashed). It is clearly seen that $aa_{\min,f}$ is well correlated with aa_{\min} ($r = 0.82$) and well reflects the increasing trend in aa_{\min} (with rising factors of $17.7/7.4$ vs. $17.7/7.3 \sim 2.4$). Both values (aa_{\min} , $aa_{\min,f}$) are well correlated with R_{\max} ($r = 0.89, 0.73$).

Figure 4b shows the values of aa_{\max} (solid), $aa_{\max,f}$ (dotted) and R_{\max} (dashed). It can also be seen that $aa_{\max,f}$ is highly correlated with aa_{\max} ($r = 0.91$) and well reflects the increasing trend in aa_{\max} (with rising factors of $25.1/18.0 \sim 1.4$ vs. $30.7/20.0 \sim 1.5$). The correlation coefficients of these two values (aa_{\max} , $aa_{\max,f}$) with R_{\max} are also very high ($r = 0.86, 0.99$).

In Model (10), the aa index is viewed as the total contributions from all the solar activities (R_z) at the present and in the past times. Therefore, the increasing trends in aa , aa_{\max} and aa_{\min} (baseline) are caused by the increasing trend in R_z (Fig. 3). This confirms the suggestions that the change in aa is caused by an increase in solar magnetic activity over the last century (Lockwood et al., 1999), and that the increasing trend in magnetic storm is most likely caused by solar activity (Clilverd et al., 1998). As the value of aa_{\min} and its increasing trend can be well reconstructed by Model (10) to-

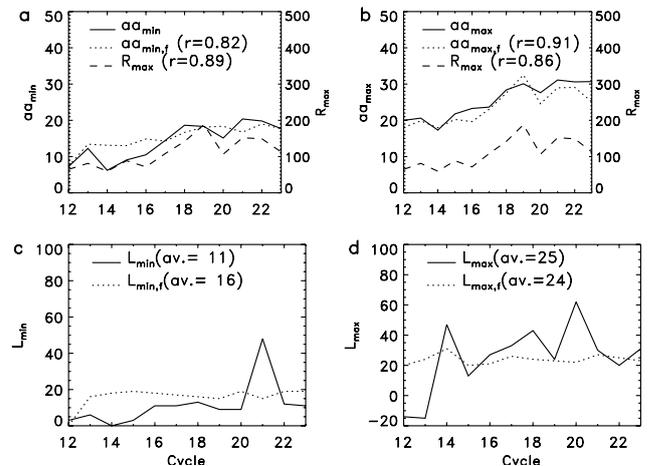


Fig. 4. The observed (a) geomagnetic minimum (aa_{\min}), (b) maximum aa (aa_{\max}), (c) lag times of aa to R_z at solar minimum (L_{\min}) and (d) at maximum (L_{\max}) for Cycles $n = 12$ – 23 are shown by solid lines, and the corresponding values ($aa_{\min,f}$, $aa_{\max,f}$, $L_{\min,f}$, $L_{\max,f}$) from the reconstructed series (Eq. 10) are shown by dotted lines in (a) and (b) show the maximum amplitude (R_{\max}) of sunspot cycle for comparison. The correlation coefficients of aa_{\min} with $aa_{\min,f}$ and R_{\max} are $r = 0.82$ and 0.89 , respectively. The correlation coefficients of aa_{\max} with $aa_{\max,f}$ and R_{\max} are $r = 0.91$ and 0.86 , respectively.

gether with the increasing trend in R_z , aa_{\min} represents the geomagnetic activity generated by earlier solar activities.

3.2 The longer lag times of aa to R_z at solar maxima than at minima

The aa index tends to lag behind R_z about 2–3 years around a solar maximum (Wang et al., 2000; Echer et al., 2004), and about 1 year around a solar minimum (Legrand and Simon, 1981; Wilson, 1990; Wang and Sheeley, 2009).

The lag times of L_{\min} (solid) and $L_{\min,f}$ (dotted) are shown in Fig. 4c, and those of L_{\max} (solid) and $L_{\max,f}$ (dotted) are shown in Fig. 4d. It is seen that the average lag time at solar minima from the reconstructed series ($\bar{L}_{\min,f} = 16$) is near to the observed one ($\bar{L}_{\min} = 11$), and that the average lag time at solar maxima from the reconstructed series ($\bar{L}_{\max,f} = 24$) is close to the observed one ($\bar{L}_{\max} = 25$). In addition, the reconstructed series can also indicate the smaller average lag time at solar minima than at solar maxima ($16 < 24$ vs. $11 < 25$). The weak correlations between the fitted values ($L_{\min,f}$ or $L_{\max,f}$) and the observed values (L_{\min} or L_{\max}) reflect the fact that aa has several solar sources (solar flares, coronal mass ejections and solar winds etc.), and that these sources peak at different times relative to R_z for each cycle and generate the geomagnetic activities with different lag times (see Discussions).

At a solar maximum, a part of aa is contributed from the solar activities (R_z) during the rising phase of the current cycle, which are weaker than the maximum value (R_{\max}). These smaller values of R_z lead the aa index not to reach its maximum at the same time of R_{\max} . After the timing of R_{\max} , a part of aa is contributed from the values of R_z around the maximum (R_{\max}), which make the aa index larger than that it should have been. So the aa index reaches its maximum at a time later than the timing of R_{\max} .

At a solar minimum, a part of aa is contributed from the solar activities (R_z) during the declining phase of the preceding cycle, which are stronger than the minimum value (R_{\min}). These larger values of R_z lead the aa index not to reach its minimum at the same time of R_{\min} . After the timing of R_{\min} , a part of aa is contributed from the values of R_z around the minimum (R_{\min}), which make the aa index smaller than that it should have been. So the aa index reaches its minimum at a time later than the timing of R_{\min} .

Because the stronger the previous R_z values, the more they contribute to the subsequent aa values, and the longer the lag time of aa to R_z (10). The values of R_z around a solar maximum are much larger than those around the preceding minimum (and R_z changes more slowly during the declining phase near the minimum than near the maximum). Therefore, the lag times of aa to R_z near solar maxima (about two years) are longer than those near solar minima (about one year). While the linear relationship between aa and R_z (Fig. 2) can not indicate the longer lag times of aa to R_z at solar maxima and the shorter lag times at solar minima. The sharp increase in lag time in cycle 14 (Fig. 4d) is mainly due to the higher maxima in Cycles 12–13 than that in Cycle 14.

3.3 The stronger correlations between aa and R_z at rising phases than at declining phases

According to Model (10), $aa(t)$ at time t comes from the total contributions of $R_z(t - \Delta t)$ at various Δt with a decay factor of $e^{-\Delta t/\tau}$. The longer the time interval of R_z preceding aa , the less its contribution.

During the rising phase of a solar cycle, the aa value is contributed from two parts of R_z : one is that during the same rising phase, and another is that during the previous declining phase (for simplicity, we neglect the less contributions from the even earlier data). The former varies approximately linearly with time in an ascending way and has a linear response to aa that makes the correlation of aa with R_z positive. The latter has a longer time delay Δt and so contributes less to aa than the former. Thus, the correlation between aa and R_z at the rising phase of a solar cycle is strong (Du, 2011b).

During the declining phase of a solar cycle, there are also two parts of R_z that contribute to aa : one is that during the same declining phase, and another is that during the preceding rising phase. The former varies approximately linearly with time in a descending way and has a roughly linear response to aa that makes the correlation of aa with R_z positive. However, the latter varies with time in an opposite ascending way which contributes a negative correlation. Because the values of R_z around a solar maximum are much larger than those around a solar minimum, the contribution of the negative correlation around the solar maximum is larger than that around the solar minimum. Thus, the correlation between aa and R_z at the declining phase of a solar cycle is weak (Du, 2011b).

3.4 The decreasing trend in the correlation between aa and R_z

The decreasing trend in the correlation between aa and R_z over time is due to the increasing trend in R_z (solar magnetic activity) over the last century (Fig. 3). The values of R_z around solar minima have no significant changes from Cycles 12 to 23, so there are no significant variations in the correlations at rising phases according to Eq. (10) and Sect. 3.3. However, the increasing trend in the values of R_z around solar maxima (R_{\max} in Fig. 4b) contributes increasing negative correlations of aa with R_z at the following declining phases. This leads to a descending trend in the correlations at declining phases, which finally makes the correlation of aa with R_z decrease for each cycle or for a given time window (Du, 2011b).

3.5 The increasing trend in the lag time of aa to R_z

The increasing trend in the lag time of aa to R_z is also due to the increasing trend in R_z over the last century. The larger the values of R_z around solar maxima, the more their effects on the aa values during the declining phases, and the longer the

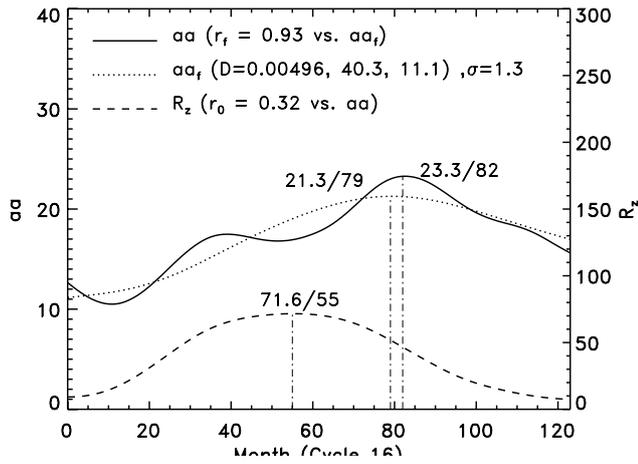


Fig. 5. Results for Cycle 16 (from April 1923 to July 1933), aa (solid), R_z (dashed), and the reconstructed series aa_f (dotted) from Model (10). The correlation coefficients of aa with R_z and aa_f are $r_0 = 0.32$ and $r_f = 0.93$, respectively. The peak sizes/time intervals (in months) of these peaks from the minimum are also labelled.

lag time of aa to R_z . So, the increasing trend in R_z around solar maxima produces an increasing trend in the lag time of aa to R_z during the declining phases (besides the decreasing correlation of aa with R_z).

3.6 The turning point of the correlation of aa with R_z around Cycle 19

The R_z values show a roughly increasing trend from Cycles 12 to 19 and a roughly declining trend since Cycle 19 (Fig. 3). This is the reasons for the decreasing trend in the correlation between aa and R_z and the increasing lag time of aa to R_z before Cycle 19 (Du, 2011b). These trends changed since Cycle 19 (around 1958).

4 The results for modelling each cycle of 12–23

In this section, Model (10) is applied to each Cycle of 12–23. Considering that the values of R_z around a solar minimum are small and generate less geomagnetic activities than during other times, and that the activities before the solar minimum will undergo longer decay times, we take the starting time in the summation of Model (10) as the timing of the previous solar minimum (t_0).

4.1 For an individual Cycle 16

Firstly, we apply Model (10) to an arbitrary solar Cycle 16 as an example. Figure 5 shows the time series of aa (solid), R_z (dashed) for Cycle 16 (from April 1923 to July 1933), and the reconstructed series (aa_f , dotted).

It is seen from Fig. 5 that the reconstructed series (aa_f) well reflects the profile of aa and the time delay of aa_{max}

Table 2. Fitted parameters of D , τ and aa_0 , and the correlation coefficients of aa with R_z (r_0) and aa_f (r_f) for Cycles $n = 12$ –23.

n	t_0	$10^3 D$	τ	aa_0	r_0	r_f
12	Jul 1878	7.10	24.0	9.4	0.56	0.83
13	May 1889	76.26	1.3	9.9	0.90	0.90
14	Aug 1901	6.62	27.6	8.0	0.55	0.92
15	Nov 1912	10.59	12.8	10.8	0.84	0.96
16	Apr 1923	4.96	40.3	11.1	0.32	0.93
17	Jul 1933	2.67	58.0	14.6	0.30	0.99
18	Dec 1943	1.20	53.2	19.9	0.17	0.70
19	Jan 1954	6.89	10.1	17.6	0.88	0.98
20	Sep 1964	1.02	137.0	16.6	-0.18	0.83
21	Feb 1976	1.35	58.5	20.6	-0.00	0.71
22	Jan 1986	5.17	15.3	19.5	0.76	0.96
23	Apr 1996	6.82	17.2	15.1	0.65	0.84
\bar{x}		10.88	37.9	14.4	0.48	0.88

(23.3) to R_{max} (71.6): $79 - 55 = 24$ vs. $82 - 55 = 27$. The three parameters in Model (10) for Cycle 16 are $D = 4.96 \times 10^{-3}$, $\tau = 40.3$ (months) and $aa_0 = 11.1$. The correlation coefficient of aa with aa_f ($r_f = 0.93$) is much higher than that ($r_0 = 0.32$) of aa with R_z . Besides, these results are not sensitive to the accurate definition of the timings of solar minima – the results remain nearly the same even if the starting time (t_0) shifts a few months forward or backward.

4.2 The fitted parameters for Cycles 12–23

For each Cycle of $n = 12$ –16, we use Model (10) to fit the aa index as done in the previous section. The results for all individual Cycles of $n = 12$ –16 are shown in Fig. 10. The three parameters are listed in Table 2 and shown in Fig. 6.

It should be noted in Fig. 6a that the parameters D (solid) and τ (dotted) tend to vary in an opposite trend: D increases while τ decreases, and vice versa. The correlation coefficient between them is $r = -0.41$ (or -0.87 if not considering the two outliers of Cycles 13 and 20). It represents the fact that the faster the energy transfer from solar activity to geomagnetic activity (via solar winds, for example) in a solar cycle, the less time the energy transfer needs. The average D and τ are $\bar{D} = 10.88 \times 10^{-3}$ and $\bar{\tau} = 37.9$ (months), respectively, implying that a solar activity will affects the geomagnetic activity in the subsequent time period of about 38 months (3 years) on average.

The values of aa_0 (Fig. 6b, shifted downward by 5 for clarity) are all positive, almost all larger than the basis value (9.9) in Fig. 3, and, most importantly, vary in an increasing trend. In Fig. 6b are also plotted the maximum amplitude of sunspot cycle (R_{max} , dashed) and the preceding aa minimum (aa_{min} , dotted). It should be noted that aa_0 is highly correlated with both aa_{min} ($r = 0.95$) and R_{max} ($r = 0.89$). The average aa_0

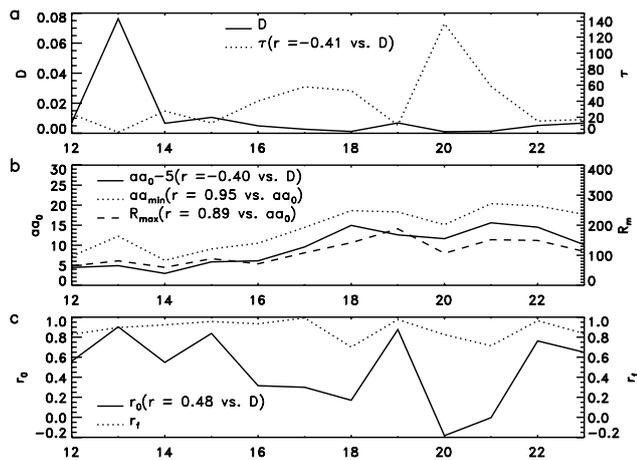


Fig. 6. Parameters for Cycles $n = 12$ – 23 : (a) for D (solid) and τ (dotted); (b) for $aa_0 - 5$ (solid), aa_{\min} (dotted), and R_{\max} (dashed). (c) The correlation coefficient of aa with R_Z (r_0 , solid) and that of aa with aa_f (r_f , dotted). The correlations of D with τ , aa_0 , and r_0 are $r = -0.41$, -0.40 , and 0.48 , respectively. The correlations of aa_0 with aa_{\min} and R_{\max} are $r = 0.95$ and 0.89 , respectively.

($\overline{aa_0} = 14.4$) is close to the average aa_{\min} ($\overline{aa_{\min}} = 14.1$). The value of aa_0 well reflects the level of and variation in aa_{\min} . These facts imply that, besides the R_Z values in the current cycle, the R_Z values in the previous cycles also generate geomagnetic activities in the current cycle, and that aa_0 , like aa_{\min} , is generated by the earlier R_Z values.

Figure 6c shows the correlation coefficient of aa with R_Z (r_0 , solid) and that of aa with aa_f (r_f , dotted) for each Cycle of $n = 12$ – 16 (Table 1). The average r_f ($\overline{r_f} = 0.88$) is much higher than the average r_0 ($\overline{r_0} = 0.48$). Therefore, Model (10) reflects the relationship between solar and geomagnetic activities in a more reliable way than using a simple linear function.

The correlation coefficients of r_0 with D and τ are $r = 0.48$ (or 0.85 if not considering Cycles 13 and 20) and -0.91 , respectively. Therefore, D reflects the linear correlation and τ reflects the nonlinear correlation of aa with R_Z . Besides, τ is well correlated with L_{\max} (0.70), and thus τ is related to the time delay of aa_{\max} to R_{\max} .

4.3 The reconstructed aa series for Cycles 12–23

Combing the results of Cycles 12–23 in the above section, the aa series from Cycles 12 through 23 can be reconstructed (aa_f), as shown in Fig. 7 (dotted line).

The standard deviation of the reconstruction is $\sigma = 1.8$. The correlation coefficient of aa (solid, shifted downward by 5 for clarity) with aa_f ($r_f = 0.95$) is higher than the previous value (0.85 in Fig. 1), and much higher than that of aa with R_Z ($r_0 = 0.61$). This means that about $r_f^2 = 90.3\%$ of the variation in aa can be explained by Model (10).

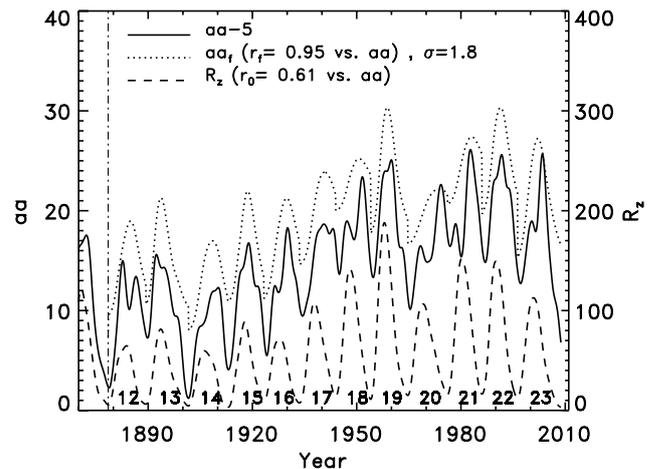


Fig. 7. Time series of $aa - 5$ (solid) and R_Z (dashed) since January 1870. The dotted line shows the reconstructed aa series (aa_f) from each Cycle of 12–23 by Model (10). The correlation coefficients of aa with R_Z and aa_f are $r_0 = 0.61$ and $r_f = 0.95$, respectively.

4.4 Correlations for even- and odd-numbered cycles

An even-numbered cycle is preferentially paired with the following odd-numbered one (Wilson, 1988), constituting a Hale cycle of even-odd cycle pairs: an odd-numbered cycle tends to be stronger than the previous even-numbered one, the so-called G–O rule (Gnevyshev and Ohl, 1948; Wilson, 1988). This rule is also applicable to parameter D : an odd-numbered cycle tends to have a larger D than the previous even-numbered one (Table 2), with only one exception of the E–O pair of Cycles 16–17. A similar rule is also applicable to parameter τ : an odd-numbered cycle tends to have a shorter τ than the previous even-numbered one (Table 2), with only two exceptions of the E–O pairs of 16–17 and 22–23.

The solar maximum (R_{\max}) is positively correlated with τ of the previous cycle ($r = 0.68$, or 0.82 if not considering Cycle 20), and reversely correlated with D of the previous cycle ($r = -0.57$, or -0.93 if not considering Cycle 13). These can explain the following phenomena.

As D reflects the linear correlation and τ reflects the nonlinear correlation of aa with R_Z , both the larger D and the shorter τ imply a stronger correlation (shorter lag time) for an odd-numbered cycle than for the previous even-numbered cycle. This is consistent with the idea of the more decays for even-numbered cycles than for odd-numbered cycles (Du, 2011a). Therefore, an odd-numbered cycle tends to have a stronger correlation of aa with R_Z and a shorter lag time of aa to R_Z than the previous even-numbered cycle (Du, 2011b). This cycle asymmetry has been noted by Stamper et al. (1999) that the solar wind speed peaks strongly in the declining phase of even-numbered cycles.

Table 3. Parameters for Hale cycles (H).

H	n	R_H	$10^3 D_H$	τ_H
7	12–13	73.1	41.68	12
8	14–15	74.1	8.61	21
9	16–17	89.9	3.82	49
10	18–19	164.9	4.01	32
11	20–21	129.2	1.19	98
12	22–23	131.0	6.00	16

4.5 Correlations for Hale cycles

The above correlations can also explain the variation in the correlation of aa with R_z for the Hale cycles. The even-odd cycle pairs can be numbered as the Hale cycles (H) such that $H = 7$ for Cycles $n = 12$ –13, $H = 8$ for $n = 14$ –15, ..., $H = 12$ for $n = 22$ –23. The averages of R_z , D and τ for H -cycles are listed in Table 3 such that $R_H(7) = [R_{\max}(12) + R_{\max}(13)]/2$, etc.

It is seen in Table 3 that an even-numbered H -cycle tends to be stronger than the previous odd-numbered one. Since $H = 8$, an even-numbered H -cycle tends to have a larger D and a shorter τ than the neighboring odd-numbered H -cycle. Therefore, an even-numbered H -cycle tends to have a stronger correlation and a shorter lag time of aa to R_z than the neighboring odd-numbered H -cycle (Du, 2011b).

5 Discussions and conclusions

It has been known that there are two main solar sources of geomagnetic activity (Legrand and Simon, 1981, 1989; Gonzalez and Tsurutani, 1987; Gonzalez et al., 2004; Venkatesan et al., 1991; Echer et al., 2004; Tsurutani et al., 2006). One source (coronal mass ejections or CMEs) has a frequency of occurrence that is in phase with the sunspot cycle while the second source (high-speed solar wind streams) is out of phase with the sunspot cycle. Feynman (1982), through analyzing the relationship between the annual aa and R_z from 1869 to 1975, decomposed aa into two equally strong periodic components: one (the “short lived” R component) associated with solar flares, prominence eruptions, and CMEs which follows the solar activity cycle and a second component (the “slowly varying” I component) associated with recurrent high speed solar wind streams which is out of phase with the solar activity cycle (Hathaway and Wilson, 2006). Legrand and Simon (1989) classified the geomagnetic activity (aa index) in four classes related to solar activity: (1) the magnetic quiet activity due to slow solar wind flowing around the magnetosphere, (2) the recurrent activity related to high wind speed solar wind, (3) the fluctuating activity related to fluctuating solar wind and (4) the shock activity due to shock events (CMEs).

A primary physical mechanism for energy transfer from the solar wind to the magnetosphere is magnetic reconnection between the interplanetary magnetic field (IMF) and the Earth’s magnetic field (Dungey, 1961; Gonzalez et al., 1994; Tsurutani et al., 1995; Zhang et al., 2007). The intensity and the orientation of the solar dipole is the source of the recurrent storms, while the size and the shape of the neutral sheet are the sources both of the quiet days and of the fluctuating activity (Simon and Legrand, 1989). However, there are no accurate expressions to clearly describe the relationship between the solar and geomagnetic activities.

The magnetic field is a crucial quantity to determine the state of the solar atmosphere and plays a key role in the formation and dynamics of solar activity. Sunspots (R_z) represent one of the most obvious manifestations of local magnetic fields on the Sun. The solar activity affects the geomagnetic activity in various complex processes from the Sun to the Earth, involved in the solar – interplanetary – magnetospheric – ionospheric couplings (Tsurutani et al., 2006). The aa geomagnetic index integrates all the effects on magnetosphere of several sources, such as solar flares, CMEs, and fast solar wind streams. The relationship between aa and R_z is not a simple linear or nonlinear function, rather it is an integral response function (10).

In this study, we presented a integral response model, $aa(t) = D \int_{t'=-\infty}^t R_z(t') e^{-(t-t')/\tau} dt' + aa_0$, to describe the relationship between the output (geomagnetic activity, aa) and the input (solar activity, R_z) of a (solar-terrestrial) system. In this model, the output $aa(t)$ depends not only on the present input $R_z(t)$, but also on past values. The output is a weighted sum of previous values of the input (Eqs. 2, 10), and the weights are given by the response function, $h(t-t') \propto e^{-(t-t')/\tau}$. The earlier the input, the less it contributes to the output. Parameter D represents the “Dynamic response factor” of aa to R_z and reflects the linear correlation of aa with R_z . Parameter τ represents the “response time scale” of aa to R_z (it needs time for the energy transfer from solar to geomagnetic activity) and reflects the nonlinear correlation of aa with R_z . Parameter aa_0 represents the geomagnetic activity generated by earlier solar activities (R_z , solar flares, CMEs, shocks, and solar winds etc.). For each solar cycle the aa_0 value reflects and is well correlated with the geomagnetic minimum (aa_{\min}).

For a linear relationship, the correlation coefficient between the aa and R_z series (3) is only $r_0 = 0.61$, implying that only about $r_0^2 = 37.2\%$ of the variation in aa can be explained by a linear dependence. However, when using our Model (10), the correlation between aa and the reconstructed series (aa_f) for the overall data (Fig. 3) is much higher ($r_f = 0.85$) than the above value, implying that about $r_f^2 = 72.3\%$ of the variation in aa can be explained by Model (10). If this model is applied to each solar cycle, the correlation will be even higher ($r_f = 0.95$), implying that about $r_f^2 = 90.3\%$ of the variation in aa can be explained by Eq. (10).

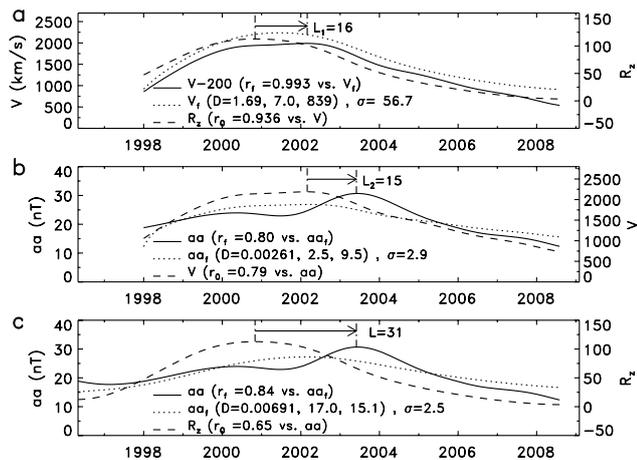


Fig. 8. (a) Monthly mean of integrated daily CME linear speed V – 200 (solid, shifted downward by 200 for clarity) and R_z (dashed) smoothed with a 24-month Gaussian filter. The dotted line shows the reconstructed series (V_f) from January 1998 to September 2008 by Model (10). The correlation coefficients of V with R_z and V_f are $r_0 = 0.936$ and $r_f = 0.993$, respectively. The lag time of V to R_z (at maximum) is $L_1 = 16$ (months). (b) Similar results for the relationship between aa (solid) and V (dashed). The correlation coefficients of aa with V and the reconstructed series aa_f are $r_0 = 0.79$ and $r_f = 0.80$, respectively. The lag time of aa to V is $L_2 = 15$ (months). (c) Similar results for the relationship between aa (solid) and R_z (dashed). The correlation coefficients of aa with R_z and the reconstructed series aa_f are $r_0 = 0.65$ and $r_f = 0.84$, respectively. The lag time of aa to R_z is $L = 31$ (months). Other numbers indicate the fitted parameters (D , τ , y_0) and σ .

This model can naturally explain in part the time delay of aa to R_z and the following phenomena.

1. The significant increase in the aa index over the twentieth century (Feynman and Crooker, 1978; Clilverd et al., 1998; Demetrescu and Dobrica, 2008; Lukianova et al., 2009), in either the cycle-averaged level or the baseline (Figs. 3, 4a, b and 6b).
2. The longer lag times of aa to R_z at solar maxima than at solar minima (Legrand and Simon, 1981; Wilson, 1990; Wang et al., 2000; Echer et al., 2004; Wang and Sheeley, 2009).
3. The stronger correlations between aa and R_z at rising phases than at declining phases (Du, 2011b).
4. The decreasing trend in the correlation between aa and R_z over time (Borello-Filisetti et al., 1992; Kishcha et al., 1999; Du, 2011b).
5. The increasing trend in the lag time of aa to R_z over time (Du, 2011b).
6. The turning point of the correlation of aa with R_z around Cycle 19 (Du, 2011b).

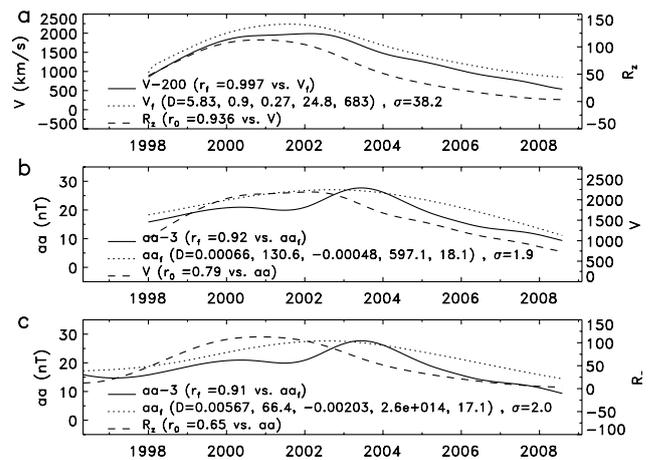


Fig. 9. Similar to Fig. 8 but using the double-decay model (12). (a) The correlation coefficient of V (solid, shifted downward by 200 for clarity) with the reconstructed series V_f (dotted) from R_z (dashed) by Eq. (12) is now $r_f = 0.997$. (b) The correlation coefficient of aa (solid, shifted downward by 3 for clarity) with the reconstructed series aa_f (dotted) from V (dashed) by Eq. (12) is now $r_f = 0.92$. (c) The correlation coefficient of aa (solid, shifted downward by 3 for clarity) with the reconstructed series aa_f (dotted) from R_z (dashed) by Eq. (12) is now $r_f = 0.91$.

7. The stronger correlations (and shorter lag times) at odd-numbered cycles than at the previous even-numbered cycles (Du, 2011b).
8. The stronger correlations (and shorter lag times) at even-numbered H -cycles than at the previous odd-numbered H -cycles (Du, 2011b).

The decreasing trend in the correlation between aa and R_z has ever been explained by the increasing occurrence of high-speed solar wind streams during the declining phase of solar cycle (Bame et al., 1976; Borello-Filisetti et al., 1992; Mussino et al., 1994; Tsurutani et al., 1995; Kishcha et al., 1999). However, during the declining phase of solar cycle, why does the occurrence of high-speed solar wind streams increase? In our Model (10), this trend is also due to the increasing trend in solar magnetic activity (R_z) over the last century (Sect. 3.1).

In this study, we used the index of sunspot number (R_z) to estimate the level of solar (magnetic) activity and its correlation with geomagnetic activity (aa). We have not specified a special activity (e.g., solar wind) to describe the interaction process of solar-geomagnetic activities. In fact, magnetic fields play a determining role in the formation and dynamics of solar activity. The magnetic field in solar corona (either close or open) is closely related to that on photosphere. Besides the sunspots, the magnetic fields may also produce other events of solar activities, such as solar flares, prominence eruptions, energetic protons, CMEs and solar winds (Legrand and Simon, 1989). Some of these events (e.g.,

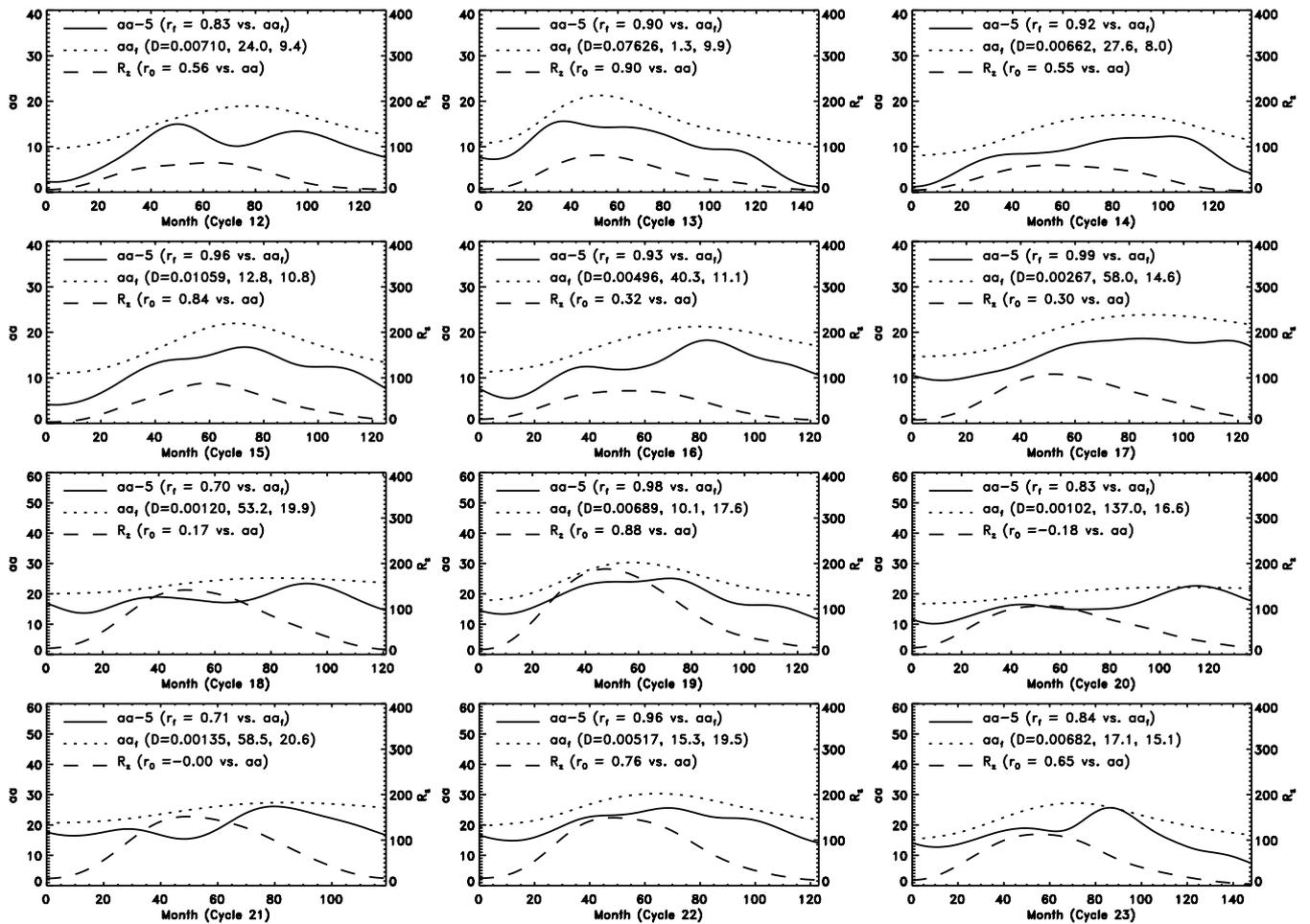


Fig. 10. Monthly mean $aa-5$ (solid, shifted downward by 5 for clarity) and R_z (dashed) smoothed with a 24-month Gaussian filter, and the reconstructed aa series (aa_f , dotted) by Model (10) for each cycle of $n = 12$ to 23 . In each plot are also shown the correlation coefficient of aa with R_z (r_0), that of aa with aa_f (r_f), and the three parameters of D , τ and aa_0 .

solar flares) are produced soon or close in time to the formation of sunspots (R_z). These events affect the geomagnetic fields (aa) in a more linear manner. Some of other events (solar winds) are produced later than the formation of sunspots, whose effect on aa is in a more nonlinear manner. These activities may be produced in different (nonlinear) processes more or less similar to Eq. (10), and play a role of mid-processes from the solar magnetic activity (R_z) through coronal magnetic activity to geomagnetic activity (aa). This may be the reasons why solar winds are later than R_z and occur often during declining phases, and why there are often more peaks in the aa index. A similar process is also applicable to the relationships between aa and these activities (solar flares, CMEs and solar winds). The effects of these processes are all integrated in Model (10).

As an example, we study the correlations of CME with solar activity (R_z) and geomagnetic activity (aa). Figure 8a shows the monthly mean of integrated daily CME linear

speed³ V (solid, shifted downward by 200 for clarity) and R_z (dashed) smoothed with a 24-month Gaussian filter (Eq. 1). The dotted line shows the reconstructed series (V_f) from January 1998 to September 2008 based on Model (10). One can see that V lags behind R_z about $L_1 = 16$ (months) at the solar maximum and that the correlation coefficient of V with the reconstructed series V_f ($r_f = 0.993$) is higher than that ($r_0 = 0.936$) of V with R_z , meaning that about 98.6% (87.6%) of the variation in V can be explained by this model (linear dependence).

Figure 8b shows the relationship between aa (solid) and V (dashed). One can also see that aa lags behind V about $L_2 = 15$ (months) and that the correlation coefficient of aa with the reconstructed series aa_f ($r_f = 0.80$) by Model (10) is higher than that ($r_0 = 0.79$) of aa with V . Figure 8c shows the relationship between aa (solid) and R_z (dashed). The correlation coefficient of aa with the reconstructed series aa_f

³http://cdaw.gsfc.nasa.gov/CME_list/

($r_f = 0.84$) by Model (10) is higher than that ($r_0 = 0.65$) of aa with R_z . It should be pointed out that the lag time of aa to R_z ($L = 31$ months) is longer than the lag times of both V to R_z (L_1) and aa to V (L_2), and equal to $L_1 + L_2$. This fact just reflects the energy transfer from solar activity (R_z) through a mid-process (CME in this case) to geomagnetic activity (aa). Similar conclusions may also hold for other solar activities (solar flares and solar winds *etc*) that have different lag times relative to R_z and that generate geomagnetic activities with different lag times relative to these activities.

As R_z and aa are only rough estimates of solar and geomagnetic activities, respectively, the geomagnetic activity (aa) is only partially predictable by the R_z index. Besides, there are two peaks in either aa or CME-related storms, one near the peak in R_z and another a few years later (Gonzalez and Tsurutani, 1987; Gonzalez et al., 1994, 2004; Tsurutani et al., 2006), while there is usually only one peak in Model (10) for a solar cycle. So this model can only describe the main profile of aa and can not predict its fine structures. The fine structures are related to short-time variations in the sources of aa (CMEs, solar winds and coronal holes etc.) and solar magnetic fields (in either solar surface or solar corona) as well. To improve the relationship between the output $y(t)$ and input $x(t)$, the following double-decay model can be used,

$$y(t) = \int_{t'=-\infty}^t x(t') \left[D_1 e^{-(t-t')/\tau_1} + D_2 e^{-(t-t')/\tau_2} \right] dt' + y_0 \quad (12)$$

$$= \sum_{t'=t_0}^t x(t') \left[D_1 e^{-(t-t')/\tau_1} + D_2 e^{-(t-t')/\tau_2} \right] + y_0.$$

The results based on Model (12) using the data of Fig. 8 are shown in Fig. 9.

It is seen in Fig. 9a that the correlation coefficient of V (solid, shifted downward by 200 for clarity) with the reconstructed series V_f (dotted) from R_z (dashed) by Model (12) is now $r_f = 0.997$, slightly higher than that (0.993) of V with the reconstructed series based on Model (10) in Fig. 8a. In Fig. 9b, the correlation coefficient of aa (solid, shifted downward by 3 for clarity) with the reconstructed series aa_f (dotted) from V (dashed) by Eq. (12) is now $r_f = 0.92$, higher than that (0.80) of aa with the reconstructed series by Model (10) in Fig. 8b. Figure 9c shows that the correlation coefficient ($r_f = 0.91$) of aa (solid, shifted downward by 3 for clarity) with the reconstructed series aa_f (dotted) from R_z (dashed) by Eq. (12) is higher than that (0.84) of aa with the reconstructed series by Model (10) in Fig. 8c.

The main profile of aa can be well predicted from the R_z series by Model (10). Even if using another more complex function, such as a geometric function of $aa(t) = D \int_{t'=-\infty}^t R_z^\gamma(t') e^{-(t-t')/\tau} dt' + aa_0$ with an additional parameter γ , there will be no significant improvement in the correlation (r_f increases from 0.85 to only 0.86). Therefore, the relationship between aa and R_z is mainly due to the integral response function. The relationship can be improved by the double-decay model (12). If considering two different lag times of y to x in Eq. (12), the result might be fur-

ther improved. The remainder might be explained by other activities, such as nonlinear Alfvén waves, cosmic rays (Nagashima et al., 1991), and the interaction (Corotating Interaction Regions or CIRs) of fast with slow solar wind streams (Simon and Legrand, 1986; Richardson and Cane, 2002; Tsurutani et al., 2006), etc.

The main points of this paper may be summarized as follows,

1. An integral response model is proposed to describe the relationship between the geomagnetic index (aa) and sunspot number (R_z): $aa(t) = D \int_{t'=-\infty}^t R_z(t') e^{-(t-t')/\tau} dt' + aa_0$. Parameters D and τ reflect the linear and nonlinear correlations of aa with R_z , respectively. Parameter aa_0 represents the geomagnetic activity generated by earlier solar activities.
2. For all data from Cycles 12 through 23, the correlation coefficient of aa with the reconstructed series based on this model ($r_f = 0.85$) is much higher than the linear correlation coefficient ($r_0 = 0.61$) of aa with R_z .
3. If this model is applied to each solar cycle, the correlation coefficient of aa with the reconstructed series is higher ($r_f = 0.95$). The aa_0 values reflects and is well correlated with the aa minimum of solar cycle.
4. This model can naturally explain in part some phenomena related to the correlation of aa with R_z , the lag time of aa to R_z , and their temporal variations.

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