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Polarization properties of Gendrin mode waves observed in the Earth's magnetosphere: observations and theory

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Abstract. We show a case of an outer zone magnetospheric electromagnetic wave propagating at the Gendrin angle, within uncertainty of the measurements. The chorus event occurred in a "minimum B pocket". For the illustrated example, the measured angle of wave propagation relative to the ambient magnetic field θ_{kB} was $58^\circ \pm 4^\circ$. For this event the theoretical Gendrin angle was 62°. Cold plasma model is used to demonstrate that Gendrin mode waves are right-hand circularly polarized, in excellent agreement with the observations.

Keywords. Electromagnetics (Electromagnetic theory) – Magnetospheric physics (Plasma waves and instabilities) – Radio science (Magnetospheric physics)

1 Method of analyses

Minimum Variance Analyses (MVAs) were performed on the chorus elements' magnetic field components in this paper. With this method, the covariance matrix is calculated and then diagonalized (Sonnerup and Cahill, 1967; Smith and Tsurutani, 1976). The eigenvectors give the directions of maximum, intermediate and minimum variances. The standard notation of B_1, B_2 and B_3 is used, respectively (see details in Smith and Tsurutani, 1976). By definition, an electromagnetic planar wave has the properties: $\nabla \boldsymbol{B} = 0$, and $\boldsymbol{k} \cdot \boldsymbol{B} = 0$, where $\boldsymbol{k}$ is the wave vector. Thus, the direction of wave propagation is the minimum variance direction (corresponding to B_3) and it is normal to the B_1-B_2 plane. The polarization of the wave is determined by the ratio λ_1/λ_2 where λ is an eigenvalue of the covariance matrix. Hodograms (in B_1 versus B_2) give a more visual perspective of the polarization (circular, elliptical or linear). An alternative method to determine wave polarization is based on multidimensional spectral analyses (Santolik et al., 2003). Their approach provides comprehensive analysis but invokes both magnetic and electric field measurements. In the current study the focus is on the magnetic polarization properties of the waves and thus the study will be restricted to magnetic field measurements alone. Thus, we chose the MVA method to analyze GEOTAIL data.

2 Observations

Electromagnetic chorus have been studied extensively in the Earth's magnetosphere (see Tsurutani and Smith, 1974, 1977; Anderson and Maeda, 1977; Koons and Roeder, 1990; Meredith et al., 2001, 2003, for chorus observational statistics). These waves are believed to be responsible for the creation of relativistic electrons in the radiation belts (Horne and Thorne, 1998) and may define the structure of the belts themselves (Horne et al., 2005). Here we focus on the basic physical properties of a distinct wave mode within the chorus frequency range.

In this paper, GEOTAIL observations are used to illustrate the property of waves in the dayside outer zone region of the magnetosphere. High-resolution magnetic field measurements from the PWI and WFC instruments (Matsumoto et al., 1994; Nagano et al., 1996) are analyzed. The PWI (plasma wave instrument) contains a 3-component search-coil with sensitivity of 1.5×10^{-5} nT/Hz$^{1/2}$. The WFC (wave form capture) receiver samples 8.7 s snapshots every 5 min between 10 Hz and 4 kHz.

Intense electromagnetic emissions were observed around 23:25:51 UT on 29 April 1993. The GEOTAIL position was (6.5, -4.3, 0.6 R_E) in GSE coordinates. GEOTAIL was at roughly noon local time in a minimum-B pocket. The Tsyganenko model (Tsyganenko, 2002) was used for reference. Chorus was observed in a low-band ($\omega < 0.5\omega_{ce}$) range, with

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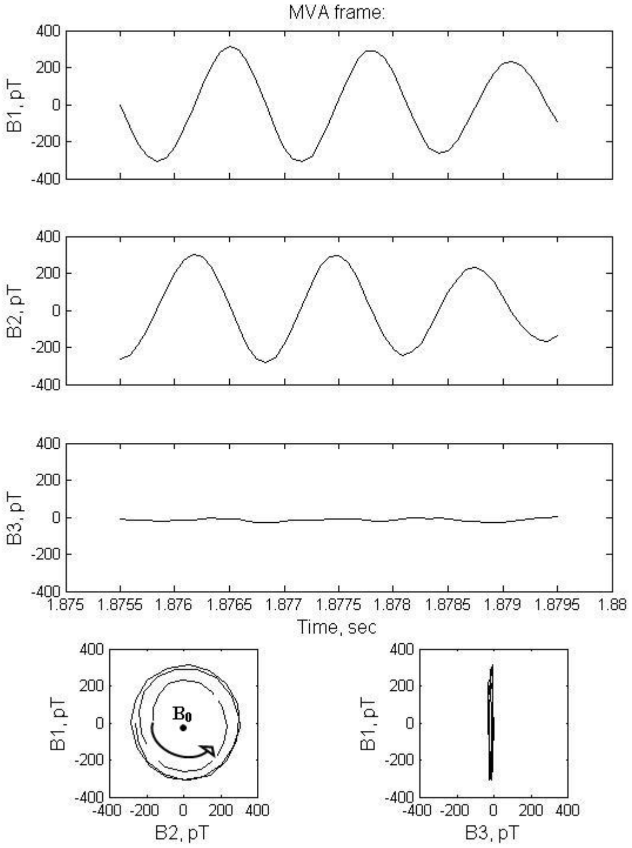

Fig. 1. Minimum variance analyses of a section of a subelement/packet. The event occurred at 23:25:51 UT on 29 April 1993. In the hodogram the direction of B_0 is indicated, which in this case is out-of-the paper. The wave is noted to be right-hand circularly polarized, planar, and propagating at $\sim 58°$ relative to B_0.

a wave frequency $f \approx 700$ Hz. In the above, ω_{ce} is the local electron cyclotron frequency. Ambient magnetic field, B_0, in GSE coordinates was (21.1, 57.4, 107.5 nT). Details on the GEOTAIL data analysis are presented in Verkhoglyadova et al. (2009).

Figure 1 shows the wave event. The top panels are 3 cycles of the wave and the B_1, B_2 and B_3 variations. The bottom two panels are the wave hodograms. The left-hand panel is the B_1-B_2 hodogram and the right-hand panel is the B_1-B_3 hodogram. The former shows that the wave is circularly polarized (the ratio λ_1/λ_2 is 1.1). The magnetic field direction is out-of-the-paper and the sense of the wave rotation is right-handed. The right-hand panel shows that the wave is plane-polarized. The wave was propagating at $\sim 58°$ relative to B_0.

There are several sources of errors in the MVA determinations. The first one is associated with the presence of random statistical noise within the magnetosphere. Tsurutani et al. (2009) estimated this isotropic noise to be ∼20 pT. For the above chorus subelement examples, the peak wave amplitudes were ∼300 pT. The noise superposed on the chorus would give an angular error of $\sim 4°$. Another source of error is instrument noise (0.02 pT/Hz$^{-1/2}$, Matsumoto et al., 1994). The lack of full wave cycles (360° of rotation) used in the analyses is a third source. These latter two errors are small compared to that of the presence of noise, so we will assume an error of $\sim 4°$ for the specific events analyzed.

It is theoretically well-known that whistler waves are circularly polarized if they propagate parallel to B_0 (Stix, 1962; Helliwell, 1965). However it is not so clear for obliquely propagating electromagnetic waves. In the next section we investigate electromagnetic whistler waves propagating in the Gendrin mode and compare their properties with the above observations.

3 Whistler mode waves and Gendrin modes

We follow the standard approach for a two-component cold plasma (Landau and Lifshitz, 1960; Krall and Trivelpiece, 1973) to study electromagnetic waves in the whistler wave frequency range between the ion cyclotron frequency (ω_{ci}) and the electron cyclotron frequency: $\omega_{ci} \ll \omega \ll \omega_{ce}$. We assume that $\omega_{ce} \ll \omega_{pe}$, where ω_{pe} is the electron plasma frequency. The dispersion relation for electromagnetic waves in this frequency range is:

$$\omega = \frac{\omega_{ce} k^2 c^2 \cos\theta}{k^2 c^2 + \omega_{pe}^2}, \tag{1}$$

where θ is the propagation angle relative to B_0. We introduce the electron inertial length, $a_e = \frac{c}{\omega_{pe}}$, and consider different limiting cases of Eq. (1). In the long-wave limit of $(ka_e)^2 \ll 1$, we obtain the classic whistler wave dispersion relation: $\omega = \omega_{ce} c^2 k^2 \cos\theta / \omega_{pe}^2$. The electron cyclotron mode $\omega = \omega_{ce}\cos\theta$ corresponds to a short-wave limit, or $(ka_e)^2 \gg 1$.

The Gendrin mode is a special mode of electromagnetic whistler waves (1) with $\omega = \omega_G = \omega_{ce}\cos\theta/2$ that exists strictly at $ka_e = 1$. It has unique propagation properties, i.e., it propagates at an angle $\theta = \theta_G$, which is called the Gendrin angle, corresponding to a minimum value of the refractive index parallel to B_0 (Gendrin, 1961). Its phase velocity is maximum among whistler waves with different k. The mode phase and group velocities are equal: $V_g = V_{ph} = \omega_G a_e$ (see also Sauer et al., 2002; Dubinin et al., 2003). The Gendrin mode is "magnetically guided" in the sense that its group velocity orthogonal to B_0 is zero. Parallel group velocity of whistler waves (1) is highest if they propagate under the Gendrin angle. Here we should note that the result by Storey (Storey, 1953; Stix, 1962) on the maximum angle of $\sim 19°28'$ between V_g and B_0 was obtained for the long-wave limit of Eq. (1) (i.e. classic whistler only) and is not applicable for Gendrin modes. According to Gendrin (1961), Gendrin modes with all frequencies ω_G (defined by θ_G) propagate with the same phase and group velocities along B_0 ($V_{g||} = V_{ph||} = \frac{\omega_{ce} a_e}{2}$).

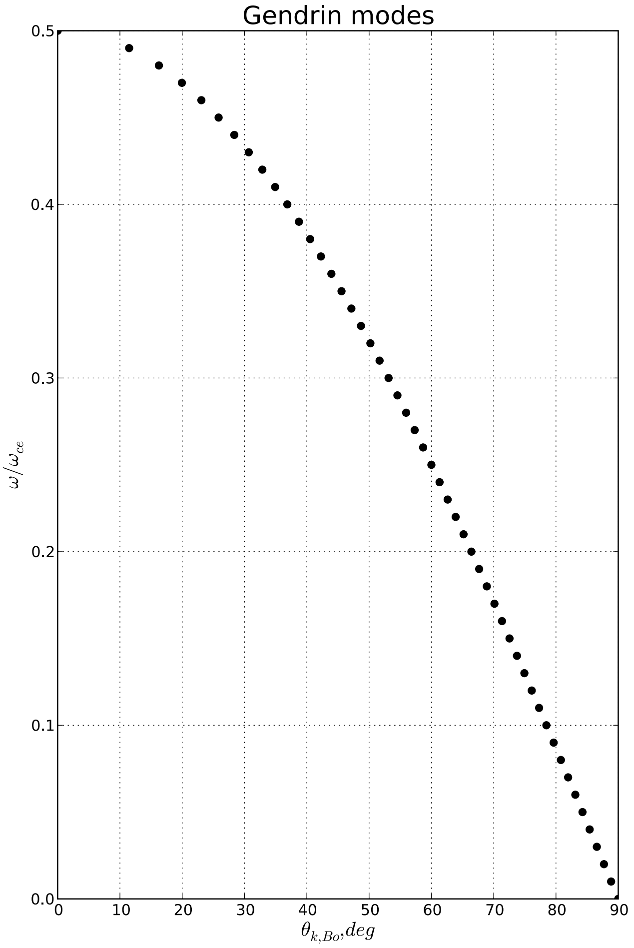

Fig. 2. Wave diagram for Gendrin modes. There is unique relationship between the propagation angle and (local) wave frequency for each of the discrete modes. Note that the mode frequency is bounded by lower-hybrid (not shown) and half-electron cyclotron frequencies.

Since the Gendrin mode frequency is defined by the propagation angle at fixed k, the mode is therefore non-dispersive. This point is illustrated by the relationship between the Gendrin mode frequency and propagation angle shown in Fig. 2. There is one mode for each value of θ_G. It should also be noted that Gendrin modes exist only in the $\omega \leq \omega_{ce}/2$ frequency range. For a lower-frequency part of the range of $\omega < 0.3\omega_{ce}$, the Gendrin modes can be highly oblique with $\theta_G > 50°$.

Since we are restricting ourselves to electron waves only, ion contributions are ignored in the dispersion (1). In other words, our results are valid for relatively fast wave processes that do not involve ion motions and the lower-hybrid frequency is a lower limit for the waves/modes considered in this paper. Namely, the frequency range for Gendrin modes is from $\omega > \omega_{ce}\sqrt{m_e/m_i} \approx 0.02\omega_{ce}$ to $0.5\,\omega_{ce}$.

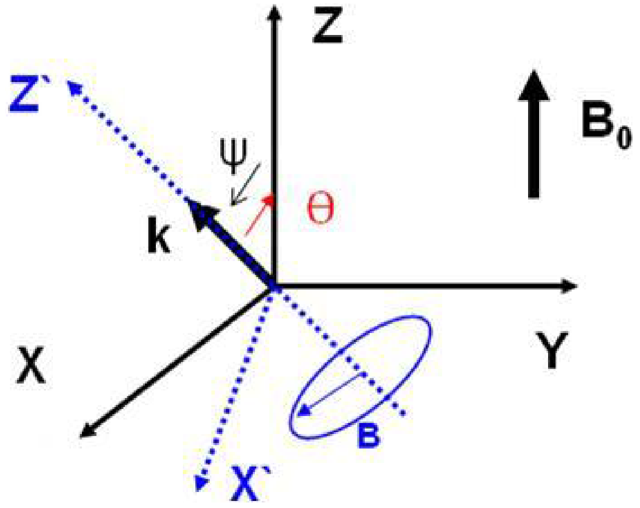

Fig. 3. Coordinate systems used. In a standard coordinate system (axes shown in black), B_0 is directed along the Z-axis and an electromagnetic wave propagates in the (XZ) plane. The MVA frame axes $\left(X'YZ'\right)$, where the new Z'-axis is aligned in the direction of wave propagation k, are shown in blue. The wave magnetic field is polarized in the $\left(X'Y\right)$ plane.

The Gendrin angle (θ_G) calculated for the above observational example from GEOTAIL is 62° whereas the measured θ_{kB} was $58° \pm 4°$. Thus, it is possible that the electromagnetic wave was propagating at the Gendrin angle within measurement uncertainties, i.e., it was a Gendrin mode wave.

4 Polarization analysis

Below, we analyze the polarization of the magnetic component B_j of the Gendrin mode from a theoretical point of view. A standard coordinate system is assumed: the background magnetic field B_0 is directed along the Z-axis and the electromagnetic wave propagates in the (XZ) plane (axes shown in Fig. 3 in black). The index j denotes components along X, Y and Z directions. Assuming $\omega_{ci} \ll \omega \ll \omega_{ce}$, the nonzero components of the Hermitian tensor of the dielectric permittivity for cold magnetized plasma (Krall and Trivelpiece, 1973; Stix, 1962) take the form:

$$\varepsilon_{xx} = \varepsilon_{yy} \approx -\frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}, \quad \varepsilon_{zz} \approx -\frac{\omega_{pe}^2}{\omega^2},$$

$$\varepsilon_{xy} = -\varepsilon_{yx} \approx -\frac{i\omega_{pe}^2\omega_{ce}}{\omega\left(\omega^2 - \omega_{ce}^2\right)}, \tag{2}$$

Using Maxwell's equations and Eq. (2) it is straightforward to find the corresponding polarization relations for a plane wave with dispersion (1):

$$B_x = -\frac{k_{||}}{k_\perp} B_z, \quad B_x = i\frac{k_{||}}{A_1\left(k_{||} - \frac{k_\perp}{A_2}\right)} B_y, \tag{3}$$

where

$$A_1 = -\frac{\frac{\omega^2}{c^2}\frac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2}+k^2}{\frac{\omega\omega_{pe}^2\omega_{ce}}{c^2\left(\omega^2-\omega_{ce}^2\right)}}, \quad A_2 = \frac{\frac{\omega_{pe}^2}{c^2}+k_\perp^2}{k_\perp k_{||}}. \tag{4}$$

Here we introduce the parallel and perpendicular (relative to B_0) components of the wave vector, $k_\perp = k\sin\theta,\ k_{||} = k\cos\theta$. For Gendrin modes, we modify Eqs. (4) and (3) by using $\omega = \omega_G$ and $k = 1/a_e$ explicitly:

$$B_x = -\frac{1}{tg\theta_G} B_z, \quad B_x = i\cos\theta_G\, B_y. \tag{5}$$

We examine the wave polarization in a corresponding MVA frame, where the new Z'-axis is aligned in the direction of wave propagation k. To find this coordinate frame, we perform a linear transformation of the coordinate system $(XYZ) \to \left(X'Y'Z'\right)$ by rotating it through an angle $-\theta_G$ (or anti-clockwise) about the Y-axis (see Fig. 3). Following Korn and Korn (1961; Eqs. 14.10–18b), we perform this transformation of the wave magnetic field $B' = \hat{T}\,B$ with the matrix:

$$\hat{T} = \begin{pmatrix} a & 0 & b \\ 0 & 1 & 0 \\ -b & 0 & a \end{pmatrix}, \quad a = \cos\theta_G,\ b = -\sin\theta_G \tag{6}$$

For the wave components we obtain with Eqs. (5) and (6):

$$\begin{aligned} B'_x &= aB_x + bB_z = iB'_y \\ B'_y &= B_y \\ B'_z &= -bB_x + aB_z = 0 \end{aligned} \tag{7}$$

Thus the Gendrin mode is right-hand circularly polarized ($B'_x = iB'_y, \quad B'_z = 0$) in a plane normal to the wave propagation direction. Note that the mode propagation can be highly oblique and the polarization plane is generally not orthogonal to B_0.

The Gendrin mode polarization was derived for a cold plasma model. Kinetic effects are negligible for this frequency range wave for spatial scales larger than electron gyro-radius. This condition is satisfied for Gendrin modes (spatial scale $\sim a_e$), assuming typical plasma parameters in the region of GEOTAIL observations.

5 Discussion and conclusion

Low-band frequency ($f < 0.5\omega_{ce}$) electromagnetic waves observed in the outer region of the Earth's dayside magnetosphere can propagate at highly oblique angles to B_0 (Goldstein and Tsurutani, 1984). We presented GEOTAIL observations showing an example of a right-hand highly oblique circularly polarized electromagnetic wave propagating at Gendrin angle within measurement uncertainties (see also Tsurutani et al., 2009). We suggest that it is a Gendrin mode wave.

The Gendrin mode is a distinct mode of electromagnetic whistler waves (Eq. 1). These modes exist only at frequencies below $\omega_{ce}/2$ and above the lower-hybrid frequency. Based on a cold plasma model, we have demonstrated theoretically for the first time that the wave magnetic field for the Gendrin mode is right-handed and circularly polarized. This theoretical result is in excellent agreement with results of the data analysis presented here, in Tsurutani et al. (2009) and in Verkhoglyadova et al. (2009).

Gendrin modes have unique propagation properties. They are non-dispersive and are "magnetically guided" so that their group velocity along background magnetic field is maximum if the wave propagates at the Gendrin angle. According to Gendrin (1961): "Thus for each frequency there is an emission angle $\theta \neq 0$, such that the beam is strictly propagated along the line of magnetic force, at a velocity independent of the frequency." Thus, these modes could be responsible for non-ducted electromagnetic wave propagation in the low-band range (Lauben et al., 2002; Helliwell, 1995). There is observational evidence that electromagnetic waves originated in the outer region of the Earth's dayside magnetosphere can propagate to the ground (Spasojevic et al., 2008). It is possible that Gendrin mode waves may be responsible for these ground observations and we suggest that further modeling efforts be undertaken to determine if this is the correct interpretation or not. However this effort is beyond the scope of the present paper.

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