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Time development of electric fields and currents in space plasmas

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Abstract. Two different approaches, referred to as Bu and Ej, can be used to examine the time development of electric fields and currents in space plasmas based on the fundamental laws of physics. From the Bu approach, the required equation involves the generalized Ohm's law with some simplifying assumptions. From the Ej approach, the required equation can be derived from the equation of particle motion, coupled self-consistently with Maxwell's equation, and the definition of electric current density. Recently, some strong statements against the Ej approach have been made. In this paper, we evaluate these statements by discussing (1) some limitations of the Bu approach in solving the time development of electric fields and currents, (2) the procedure in calculating self-consistently the time development of the electric current in space plasmas without taking the curl of the magnetic field in some cases, and (3) the dependency of the time development of magnetic field on electric current. It is concluded that the Ej approach can be useful to understand some magnetospheric problems. In particular, statements about the change of electric current are valid theoretical explanations of change in magnetic field during substorms.

Keywords. Magnetospheric physics (Curent systems; Electric fields; Storms and substorms)

1 Introduction

Some space plasma researchers are keenly aware of a seemingly everlasting controversy on the "correct" approach in developing theoretical understanding of magnetospheric phenomena. On the offensive side is the Bu approach that claims magnetic field ($\boldsymbol{B}$) and plasma bulk flow ($\boldsymbol{u}$) are the primary quantities from which current density and electric field should be derived. Advocates of this approach claim it to be the *only* proper way to address magnetospheric problems (e.g. Parker, 1996; Vasyliunas, 2001, 2005). On the defensive side is the Ej approach that adopts the electric field ($\boldsymbol{E}$) and electric current density ($\boldsymbol{j}$) as the basic quantities to gain insights into the underlying physics (e.g. Alfvén, 1977; Lui, 1996; Parks, 2005; Yoon and Lui, 2006). Practitioners of this approach state that both the Bu and Ej approaches have merits and limitations, and which one is the better approach should depend on the magnetospheric phenomenon to be investigated. Insisting on only one approach would only stifle innovative thinking in scientific pursuits (Lui, 2000). Rather uncomplimentary remarks have been made by the Bu advocates to the Ej practitioners, such as referring to the Ej approach in understanding substorms as producing the "dark ages" in magnetospheric physics (Axford, 1994) and describing the Ej approach to estimate the dipolarization time scale with the current reduction time scale as "naive expectation" (Vasyliunas, 1996).

Recently, the time development of electric current is examined by Vasyliunas (2005). The main points stated there are: (1) on time scales longer than the electron plasma period, $\boldsymbol{j}$ should *only* be determined by curl of $\boldsymbol{B}$; (2) on similar time scales, $\boldsymbol{E}$ should *only* be determined by the generalized Ohm's law; and (3) substorm theory of current disruption is merely a description of change in magnetic field and not an explanation. In this paper, we examine the logic behind these statements and identify their problems. We also provide counter examples to invalidate these assertions. The SI unit will be used throughout this paper.

2 Logic behind the criticisms from the Bu approach

It is useful to recount the main logical steps leading to the conviction that $\boldsymbol{j}$ should be determined only by $\boldsymbol{B}$ through Ampere's law (Vasyliunas, 2005). In the nonrelativistic approximation when the gravitational force can be neglected, the time development of current density is given by Eq. (10.110) in Rossi and Olbert (1970)

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$$\partial \boldsymbol{j}/\partial t = \sum_a \left[-(q_a/m_a)\nabla \bullet \mathbf{K}_a + \left(q_a^2 n_a/m_a\right)(\boldsymbol{E} + \boldsymbol{U}_a \times \boldsymbol{B}) \right] + (\delta \boldsymbol{j}/\delta t)_{\text{coll}}, \tag{1}$$

where q_a, m_a, n_a, $\boldsymbol{U}_a$, and $\mathbf{K}_a$ are the charge, mass, number density, bulk velocity, and kinetic tensor, respectively, of species a, and $(\delta \boldsymbol{j}/\delta t)_{\text{coll}}$ represents the sum of all collision effects. The kinetic tensor is related to the velocity distribution function $f_a(\boldsymbol{x}, \boldsymbol{v}, t)$ of species a by

$$K_{a,ik} = m_a \int v_{a,i} v_{a,k} f_a(x, v, t) d^3 v = P_{a,ik} + n_a m_a U_{a,i} U_{a,k}, \tag{2}$$

and is consisted of the thermal and dynamic pressure tensors of species a. If one denotes all the indirect electric field terms by $\boldsymbol{R}$, i.e.,

$$\boldsymbol{R} = \sum_a \left[(q_a/m_a)\nabla \bullet \mathbf{K}_a - \left(q_a^2 n_a/m_a\right) U_a \times \boldsymbol{B} \right] - (\delta \boldsymbol{j}/\delta t)_{\text{coll}}, \tag{3}$$

and notes that the electron plasma frequency (ω_{pe}) is much higher than the ion plasma frequency by the square root of the ion-to-electron mass ratio, one obtains the approximate expression

$$\partial \boldsymbol{j}/\partial t \approx \omega_{pe}^2 \varepsilon_0 (\boldsymbol{E} - \boldsymbol{R}) \,. \tag{4}$$

With Maxwell's equations, one can show readily that

$$\mu_0 \partial \boldsymbol{j}/\partial t = -\left[\nabla \times (\nabla \times \boldsymbol{E}) + \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} \right] \tag{5}$$

where c is the light speed in vacuum. Combining Eqs. (4) and (5), one arrives at

$$\boldsymbol{E} - \boldsymbol{R} \approx -l_s^2 \nabla \times (\nabla \times \boldsymbol{E}) - \frac{1}{\omega_{pe}^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}\,, \tag{6}$$

where l_s is the plasma skin depth. At this point, an order of magnitude estimate is invoked. If L_R and T_R denote, respectively, the length and time scales of the variation of $\boldsymbol{R}$, then it follows that $\boldsymbol{E} \approx \boldsymbol{R}$ if $L_R \gg l_s$ and $T_R \gg \omega_{pe}^{-1}$. This conclusion made by Vasyliunas (2005) is indeed correct. However, several strong assertions extrapolated from this conclusion were also made:

(1) On all scales longer than the electron plasma oscillations, neither the time evolution of $\boldsymbol{j}$ nor that of $\boldsymbol{E}$ could be calculated directly. Instead $\boldsymbol{E}$ should be determined by the plasma dynamics through the generalized Ohm's law and $\boldsymbol{j}$ should be determined by $\boldsymbol{B}$ through the Ampere's law.

(2) There is no equation from which the time evolution of electric current could be calculated independently of $\nabla \times \boldsymbol{B}$.

(3) For substorm theories, statements about the change (disruption, diversion, wedge formation) of electric current are merely descriptions of change in magnetic field and not explanations.

3 Limitations of the Bu approach

There are limitations of the Bu approach on the time development of $\boldsymbol{j}$ and $\boldsymbol{E}$ that may not be apparent in previous articles expounding this approach. However, they are vitally important and should be recognized if one adopts the Bu approach to solve any magnetospheric problem.

3.1 Assumption in the equation of state

Let us first ponder on the statement that $\boldsymbol{E}$ should only be determined by the plasma dynamics through the generalized Ohm's law. Notice that this is not a statement about the time development of $\boldsymbol{E}$ because in order to do so, one has to determine the time development of plasma dynamics. Here lies a problem. In order to obtain the time development of the bulk parameters such as $\boldsymbol{U}_a$ and $\mathbf{K}_a$ in the generalized Ohm's law to obtain the time development of $\boldsymbol{E}$, one needs to go back to the velocity moments of the full Boltzmann-Maxwell's system of equations. It is well known for this approach that even with the inclusion of all terms in the generalized Ohm's law, this set of equations do not form a closed set of solvable equations unless an equation of state is assumed. Therefore, the time development of plasma dynamics (and $\boldsymbol{E}$ as a result) depends on the assumed form of the equation of state. In some cases, the form can be assumed with confidence based on some physical insights, e.g. isothermal or adiabatic process. However, observations indicate that plasmas in the real magnetosphere in general do not obey a simple equation of state, including the well-known CGL equation (Chew et al., 1956). There is at least one good underlying reason for this departure. A dynamic region of the magnetosphere does not usually form a closed system because it typically exhibits significant heat flux transport. Particle loss through precipitation into the ionosphere and/or escape along the magnetic field to the distant magnetotail or its open boundary are often non-negligible.

As a case in point, Erickson and Wolf (1980) demonstrated that the near-Earth plasma pressure expected from the steady state magnetospheric convection is too high to be compatible with observations. Precipitation and other loss processes play a significant role in relieving the near-Earth plasma pressure arising from convection (e.g. Kivelson and Spence, 1988). As a result, the equation of state associated with magnetospheric convection deviates considerably from the expectation of adiabatic convection, $P/n^\gamma = \text{constant}$ (P is the plasma pressure, n is the number density, and γ is the ratio of specific heats). Lui and Hamilton (1992) evaluated observationally various forms of the equation of state, including one based on the conservation of magnetic moment $P_\perp/(nB) = \text{constant}$ ($P_\perp$ is the plasma pressure perpendicular to the local magnetic field B), and found none to be satisfactory.

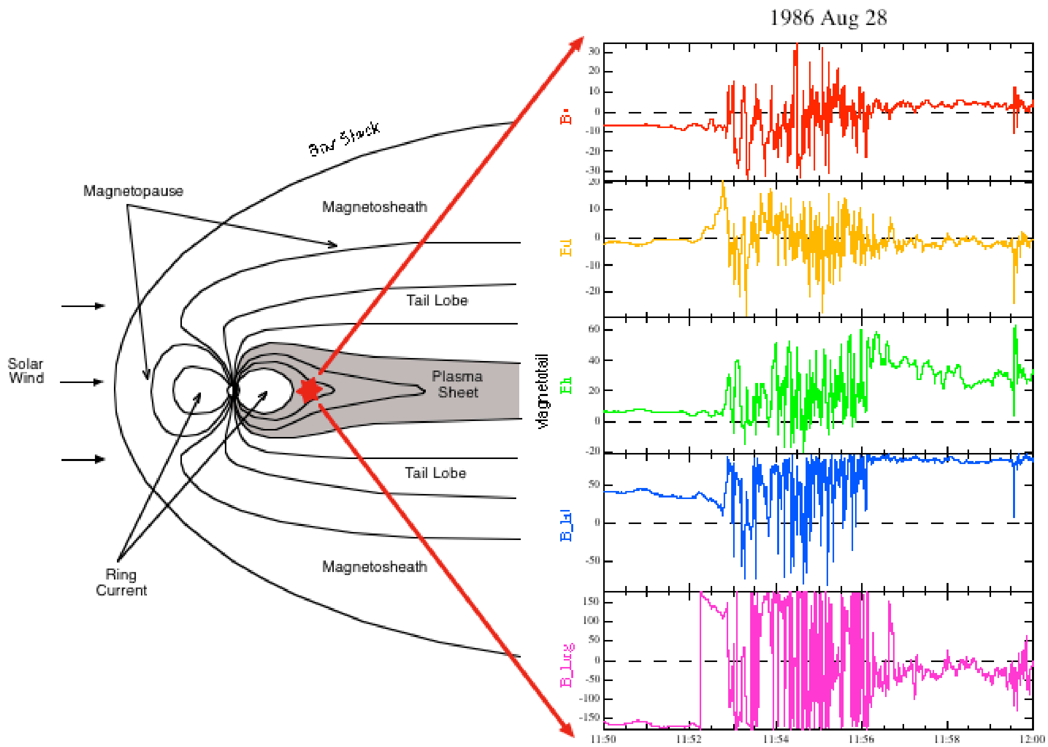

Fig. 1. (Left) Schematic illustration of current disruption in the near-Earth magnetotail. (Right) Observation of large magnetic fluctuations in all components, which are distinct characteristics of the current disruption phenomenon. The top three panels show the magnetic field components and the bottom two panels show the orientation angles of the magnetic field. The onset of large magnetic fluctuations preceded the ground substorm expansion onset by ∼1 min.

3.2 Impasse in analysis of multiscale phenomena

Can the Bu approach solve self-consistently multiscale phenomena such as the substorm expansion onset problem? During substorm onsets, large magnetic fluctuations identified as manifestation of the current disruption phenomenon can be detected in localized regions of the near-Earth magnetotail. One such example is shown in Fig. 1, taken from the AMPTE/CCE measurements. As indicated, there is a significant change in the *Bh* component over the time scale of ∼3 min. On top of this change, there are large fluctuations ($\delta B/B>1$) down to the sampling frequency. This is intrinsically a multiscale phenomenon. If one attempts to solve this problem using the generalized Ohm's law approach, then one arrives at

$$\partial\langle \boldsymbol{j}\rangle/\partial t = \sum_a \Big[-(q_a/m_a)\nabla\bullet\langle \mathbf{K}_a\rangle + \left(q_a^2 n_a/m_a\right)(\langle \boldsymbol{E}\rangle + \langle \boldsymbol{U}_a\rangle\times\langle \boldsymbol{B}\rangle) + \left(q_a^2/m_a\right)(\langle \delta n_a \delta \boldsymbol{E}\rangle + \langle \delta(n_a\boldsymbol{U}_a)\times\delta\boldsymbol{B}\rangle)\Big] + \delta\langle \boldsymbol{j}\rangle/\delta t_{\mathrm{coll}} \quad (7)$$

by writing a quantity X as $X+\delta X$ with $<\delta X>=0$. The angle bracket denotes averaging of quantities over the time-scale of the slow variations for particle parameters in the Bu approach. This equation is unsolvable without detailed kinetic calculation of the various correlation (cross product) terms even with the neglect of the inertial term on the left hand side (LHS) of Eq. (7). This impasse indicates that the approach using the generalized Ohm's law is not appropriate to address the current disruption phenomenon in which short time variations of parameters play a significant role in the plasma dynamics. The self-consistent kinetic approach in solving the problem described later in the Sect. 4.3 does not have this impasse. Recently, Yoon and Lui (2006) have derived analytical expressions for the correlation terms for some specific instabilities by solving the appropriate Boltzmann-Maxwell's system of equations.

In the current disruption model of substorm expansion onset (Lui, 1991), current disruption is a phenomenon similar to snow avalanche (Lui et al., 2000). The onset process is a small-scale kinetic one. However, in its excitation, it modifies the local plasma parameters in the surrounding region to instigate further development of the process nearby. Therefore, from this consideration, the onset process is not a large-scale one in spite of the fact that substorm expansion disturbances cover a large-scale region in both the magnetotail and the ionosphere. This is analogous to the consideration of the physical process responsible for snow avalanche. Similarly, a domino effect is not due to a large-scale disturbance but a small-scale one that cascades into a large-scale change, which is an example of an inverse cascade process in nature.

3.3 Recapitulation

The above discussion indicates that the determination of the time development of $\boldsymbol{E}$ and $\boldsymbol{j}$ through the Bu approach is not as straightforward and precise as it may appear to be. More specifically, one cannot obtain the time development of $\boldsymbol{E}$ exactly through the use of the generalized Ohm's law without an approximation, which may have questionable validity. For multiscale problems, terms involving cross products of fluctuations of plasma and field parameters in the generalized Ohm's law cannot be calculated without detailed knowledge of kinetic processes responsible for these fluctuations. This poses an impasse in solving multiscale phenomena through the Bu approach. Additional limitations of this approach in solving plasma dynamics have been discussed previously by Lui (2000).

4 Viewpoint from the Ej approach

While we do not dispute the Bu approach in solving a given problem outlined in Sect. 2 for some space phenomena may be appropriate, we do dispute that it is the *only approach or always the practical one* to obtain a self-consistent solution to all magnetospheric problems. Furthermore, the fact that $\boldsymbol{E}\approx\boldsymbol{R}$ if $L_R \gg l_s$ and $T_R \gg \omega_{pe}^{-1}$ does not imply $\partial\boldsymbol{j}/\partial t \approx 0$ since the difference between $\boldsymbol{E}$ and $\boldsymbol{R}$ in Eq. (4) is amplified by a factor of $\omega_{pe}^2\varepsilon_0$, which can be as large as $\sim 10^6$ mho/s for $n_e \sim 1\,\mathrm{cm}^{-3}$. Therefore, $\partial\boldsymbol{j}/\partial t$ can be non-negligible even when the difference between $\boldsymbol{E}$ and $\boldsymbol{R}$ may seem to be small.

4.1 Scaling in kinetic process for substorm expansion onset

If a kinetic process is important in substorm problems, then the temporal and spatial scales of the kinetic process are relevant to estimate the scales involved in Eq. (6). If we use the cross-field current instability as an example, linear dispersion analysis indicates that the excited waves have a broad frequency range and a correspondingly broad wavenumber range (Lui et al., 1991). This instability contains the lower hybrid drift instability (LHDI) at the high latitude of the current sheet. It also contains the modified two-stream (MTS) and ion Weibel instabilities close to the current sheet center (Yoon et al., 2002; Lui, 2004). For the LHDI, which is favorably excited at the plasma sheet boundary where the density gradient is high, the unstable waves at the linear stage have wavenumbers $k\rho_e \sim 1$, where ρ_e is the electron gyroradius. In other words, the scale length associated with this instability L_R is of the order of ρ_e. For the case of the Earth's plasma sheet boundary in the magnetotail, the typical electron temperature T_e is ~ 1 keV and the magnetic field B is ~ 20 nT. The spatial scale is then $L_R \sim \rho_e \sim 3.8$ km. At the same location, the number density n_e is $\sim 0.1\,\mathrm{cm}^{-3}$, giving the electron inertial length (or plasma skin depth) l_s to be ~ 16.8 km. Clearly, this environment has $l_s > L_R$, just the opposite to the conditions invoked by Vasyliunas (2005) to discredit current reduction by the cross-field current instability. Similarly, for MTS with $B \sim 5$ nT, propagation angle $\theta \sim 85°$, and wavenumber $k\rho_e \sim 0.1$, the scale length is $L_R \sim 13.2$ km $< l_s$, again not satisfying the conditions assumed by Vasyliunas (2005). In essence, the kinetic process in a current sheet with thickness of ion gyroradius can have scales much shorter than the current sheet thickness. Although the above discussion pertains only to the spatial scale rather than the time scale, both criteria on the time and spatial scales have to be satisfied in order to justify the claim by Vasyliunas (2005).

Quasilinear calculation of the cross-field current instability shows the reduction in velocity of ions and electrons to be (Lui et al., 1993; Yoon and Lui, 1993)

$$\frac{dU_i}{dt} = \frac{2e^2}{m_i}\frac{U_i}{T_{i||}}\int_0^\infty dk \frac{\gamma_k \delta B_k^2}{c^2k^2}\frac{Re\left[Z'(i\gamma_k/ku_{i||})\right]}{1+\left|D_{xy}/D_{xx}\right|^2}, \tag{8}$$

$$\frac{dU_e}{dt} = \frac{2e^2}{m_e}\frac{U_e}{T_{e||}}\int_0^\infty dk \frac{\gamma_k \delta B_k^2}{c^2k^2}\frac{Re\left[Z'(i\gamma_k/ku_{e||})\right]}{1+\left|D_{xy}/D_{xx}\right|^2}. \tag{9}$$

Here, γ_k and δB_k denote, respectively, the growth rate and the magnetic perturbation amplitude of the mode with wavenumber k. D_{xx} and D_{xy} are the *xx*- and *xy*-elements of the dispersion tensor. The function Z' is the derivative of the Fried and Conte's plasma dispersion function and Re[Z'] refers to the real part of the complex quantity Z'. The temperature is denoted by T. Subscripts i and e refer to ions and electrons, respectively. The subscript || refers to the component parallel to the local magnetic field. Note the distinction between the bulk speed U and the thermal speed u. These expressions allow an estimate to be made for the instability effect on current density reduction from $dj/dt = e[d(n_iU_i)/dt - d(n_eU_e)/dt]$. This is only an estimate though, since it is a 1-D local calculation and a quasi-linear one as well, not including fully nonlinear interactions (Lui et al., 1993; Yoon and Lui, 1993). In the next subsection, we shall discuss a procedure to determine the time evolution of $\boldsymbol{j}$ self-consistently in the nonlinear regime.

Since substorm expansion covers a large-scale region, one may be attempted to disregard a kinetic process as a potential substorm process by the reason that a kinetic process with localized disturbance has no large-scale effect in the magnetosphere and ionosphere. Two counter arguments can be made. One is that conditions conducive for the excitation of the kinetic process may exist over an extended region, bringing large-scale changes by activities from multiple sites. For example, Shinohara et al. (2001) examined the nonlinear effect of LHDI of a thin Harris current sheet (Harris, 1962) with 2-D particle simulation. As discussed earlier, LHDI onset produces variations with scales smaller than the electron inertial length. In spite of this small-scale activity, the simulation result shows that the LHDI grows rapidly at the current sheet boundary with generation of electron vortices, reduction of the current density, and modification of the magnetic field configuration. The current sheet profile is altered so much by the LHDI activity that instabilities that would not occur in the original current sheet profile become excited.

The other counter argument is that the localized region may act as an essential valve for a process that can achieve large-scale changes. Magnetic reconnection is known to be capable of producing large-scale changes. However, its existence requires the presence of a diffusion region that allows the breakdown of the frozen-in condition. This diffusion region is generally perceived to be in the small scale of electron inertial length (e.g. Vasyliunas, 1975; Treumann et al., 1995). Thus, the small-scale physical process residing in the diffusion region acts as a control valve to allow the large-scale magnetic reconnection process to exist.

4.2 Time development of current density independent of $\nabla\times\boldsymbol{B}$

In this subsection, we address the claim that the time development of $\boldsymbol{j}$ cannot be calculated independently from $\nabla\times\boldsymbol{B}$. It should be borne in mind that the Ej approach is essentially a kinetic one with fundamental physics based on individual particle motion and Maxwell's equations. For this approach, $\boldsymbol{j}$ can be calculated simply based on its definition, i.e., summing up the velocity of all particles weighted by their respective charges

$$\boldsymbol{j}(\boldsymbol{r},t)=\sum_i q_i\boldsymbol{v}_i(\boldsymbol{r},t)\,. \tag{10}$$

The summation is taken over all particles, which has a different meaning from the summation in Eq. (1) that sums over particle species. The time development of the individual particle velocity $\boldsymbol{v}_i$ is in turn governed by the electric and magnetic fields acting on the particle

$$d\boldsymbol{v}_i/dt=(q_i/m_i)\,[\boldsymbol{E}(\boldsymbol{r},t)+\boldsymbol{v}_i\times\boldsymbol{B}(\boldsymbol{r},t)] \tag{11}$$

Particles move to new locations with updated velocities at a later time t' as a result, giving a new value of $\boldsymbol{j}(\boldsymbol{r},t')$, thus determining the time development of $\boldsymbol{j}$ independent of $\nabla\times\boldsymbol{B}$. This procedure leads to a self-consistent solution of the problem because $\boldsymbol{B}$ is updated by solving the Faraday's law

$$\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t=-\nabla\times\boldsymbol{E}(\boldsymbol{r},t) \tag{12}$$

and $\boldsymbol{E}$ is updated for a 1-D problem by solving the Poisson's equation

$$\nabla\bullet\boldsymbol{E}(\boldsymbol{r},t)=\sum_i q_i\delta\,(\boldsymbol{r}-\boldsymbol{r}_i)/\varepsilon_0 \tag{13}$$

The new field values are then used to advance the particle parameters at a subsequent time, completing the cycle of self-consistent calculation. In this case, the time development of the electric current is clearly obtained independent of $\nabla\times\boldsymbol{B}$. Alternatively, to demonstrate there is more than one way to solve a given problem, one may update $\boldsymbol{E}$ by solving the Ampere's law

$$\partial\boldsymbol{E}(\boldsymbol{r},t)/\partial t=c^2\,[\nabla\times\boldsymbol{B}(\boldsymbol{r},t)-\mu_0\boldsymbol{j}(\boldsymbol{r},t)] \tag{14}$$

Note that the difference between $\nabla\times\boldsymbol{B}$ and $\mu_0\boldsymbol{j}$ is amplified by c^2 to determine the temporal change of $\boldsymbol{E}$. In the Earth's magnetotail, strong current density is typically ∼10's nA/m^2. If the difference between $\nabla\times\boldsymbol{B}$ and $\mu_0\boldsymbol{j}$ is merely 0.01%, then $\partial\boldsymbol{E}/\partial t\sim100$ mV/m/s, which is a huge change in the electric field in the magnetotail. Therefore, the term on the LHS of Eq. (14), which is the displacement current contribution to $\nabla\times\boldsymbol{B}$, should not be neglected even when the Alfvén speed is much less than the light speed in vacuum, contrary to common assumption made by many magnetospheric researchers. The sensitivity of $\partial\boldsymbol{E}/\partial t$ on the slight difference between $\nabla\times\boldsymbol{B}$ and $\mu_0\boldsymbol{j}$ in the Bu approach is a major reason why it avoids this procedure. However, it does not mean that it cannot be done.

An important point emerged from the above discussion is that the time development of $\boldsymbol{B}$ from this approach depends on $\boldsymbol{j}$ through solving for $\boldsymbol{E}$. Consequently, a theory for the temporal change of $\boldsymbol{j}$ is a valid theoretical explanation for the temporal change of $\boldsymbol{B}$.

4.3 Procedure in particle simulation to calculate the time development of current density

The above point is best illustrated with a concrete example. Nonlinear dynamics in current sheets is a hotly pursued topic in space plasma research. Particle-in-cell simulations are often used. Let us discuss the procedure used by particle simulation to investigate this nonlinear problem. For simplicity, we eliminate in the following discussion the intricacies involved in ensuring a stable numerical scheme. Typically, the simulation box is initialized with a given equilibrium. The reference frame is usually chosen such that $\boldsymbol{E}$ vanishes everywhere. For instance, for Harris current sheet, we can choose the frame of reference such that U_{iy}/T_i=$-U_{ey}/T_e$. The y-coordinate refers to the direction of $\boldsymbol{j}$ for the Harris current sheet. The initial equilibrium in this case is

$$\nabla\times\boldsymbol{B}(\boldsymbol{r},t=0)=\mu_0\boldsymbol{j}(\boldsymbol{r},t=0), \tag{15}$$

and it follows from Eq. (14)

$$\partial\boldsymbol{E}(\boldsymbol{r},t=0)/\partial t=0. \tag{16}$$

For advancing the simulation in time (Δt), the first step in the particle simulation is to update the particle parameters using the equation of motion (Eq. 11). This update gives $\boldsymbol{j}(\boldsymbol{r},t=\Delta t)\neq\boldsymbol{j}(\boldsymbol{r},t=0)$ due to the thermal fluctuations inherent in the thermal particle distribution. Updating of particle parameters is followed by advancing the field values using Eqs. (12) and (13) or (14). Since $\boldsymbol{E}(\boldsymbol{r}, t{=}0){=}0$, therefore from Eq. (12)

$$\boldsymbol{B}(\boldsymbol{r}, t = \Delta t) = \boldsymbol{B}(\boldsymbol{r}, t = 0), \tag{17}$$

and

$$\nabla \times \boldsymbol{B}(\boldsymbol{r}, t = \Delta t) = \nabla \times \boldsymbol{B}(\boldsymbol{r}, t = 0) = \mu_0 \boldsymbol{j}(\boldsymbol{r}, t = 0), \tag{18}$$

i.e. $\boldsymbol{B}$ is unchanged at this time step based on Eq. (12). Next, the new values for $\boldsymbol{E}$ are computed based on Eq. (13) or Eq. (14), yielding $\boldsymbol{E}(\boldsymbol{r},t{=}\Delta t) \neq \boldsymbol{E}(\boldsymbol{r},t{=}0)$ due to thermal displacements of the particles for the next computation of change in $\boldsymbol{j}$ by Eq. (11). Several notable features emerge from the above illustration. First, the change in the current density is obtained from the equation of motion, independent of $\nabla\times\boldsymbol{B}$. Second, the non-zero $\boldsymbol{E}(\boldsymbol{r},\Delta t)$ arises from an imbalance of charge density due to the electric current flow, which is changed from the initial value due to thermal fluctuations associated with the particle distribution. Third, the time scale of non-negligible change (Δt) can be shorter than the period for electron plasma oscillation. Indeed, explicit particle simulations are usually done with a time step value of a small fraction of the electron plasma period. Fourth, if the system is unstable to a certain kinetic instability, $\boldsymbol{E}(\boldsymbol{r},t)$ will be amplified progressively by the onset of this instability. This reflects the true nature of spontaneous excitation of a kinetic instability arising from the thermal noise. If the kinetic instability is a current-driven one, $\boldsymbol{j}$ is the free energy source. Energy conservation will ensure that unstable waves will be excited at the expense of the energy associated with $\boldsymbol{j}$, thus reducing $\boldsymbol{j}$ as the instability develops.

Recently, a 2-D particle-in-cell simulation of the cross-field current instability has been performed on a Harris-like current sheet (Lui, 2004). Reduction of current density near the sheet center, current filamentation, and highly fluctuating electric fields are seen at the nonlinear stage of the instability development.

5 Summary and conclusions

Several strong criticisms have been made against the Ej approach in addressing the time development of electric fields and currents in plasmas. We first outline the logic behind the strong criticisms by the Bu approach. We identify limitations of the Bu approach in obtaining exact solutions for the time development of electric fields and currents. We show the approximations used in reaching these criticisms are not applicable to the theory for current disruption in substorms. We then proceed to demonstrate that the time development of current density can be self-consistently calculated without the use of the curl of magnetic field and that the time development of magnetic field depends on the current density.

While we acknowledge that the Bu approach can be used to solve some magnetospheric problems, we dispute that it is the *only self-consistent* approach to solve *all* magnetospheric problems, in particular, the substorm expansion onset problem. The statement that the time development of current density could not be calculated independently of $\nabla\times\boldsymbol{B}$ stamps from the failure to recognize the legitimacy of a different approach to solve the problem based on the same fundamental physical laws. This failure has also led to the strong criticism relating to substorm expansion onset that statements about the change (current disruption, diversion, wedge formation, etc.) of the electric current are merely descriptions of change in the magnetic field and not explanations. This criticism is related to the different views on the nature of the substorm expansion onset process. The Bu approach assumes the substorm expansion onset to be a large-scale one acting perhaps on a single site. On the other hand, in the current disruption substorm model, the process is viewed as a small-scale one acting on many sites spreading over a large region and over a long time scale compared with the short time scale of the process itself. There may even be more than one single physical process involved for the observed change in electric current at substorm expansion.

In closing, it is hoped that future substorm research would focus on identifying the physical process for substorm expansion rather than quibbling over which approach is the "correct" one in determining the time development of electric fields and currents in substorms. Both Bu and Ej approaches have their own merits and limitations (Lui, 2000). It is not inconceivable that a superior substorm model would emerge from combining the strengths of these two approaches.

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