Annales Geophysicae, 23, 1433–1439, 2005
SRef-ID: 1432-0576/ag/2005-23-1433

Dispersion properties of ducted whistlers, generated by lightning discharge

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Received: 14 October 2003 – Revised: 21 February 2005 – Accepted: 3 March 2005 – Published: 3 June 2005

Abstract. Whistler-mode wave propagation in magnetospheric ducts of enhanced cold plasma density is studied. The case of the arbitrary ratio of the duct radius to the whistler wavelength is considered, where the ray-tracing method is not applicable. The set of duct eigenmodes and their spatial structure are analysed and dependencies of eigenmode propagation properties on the duct characteristics are studied. Special attention is paid to the analysis of the group delay time of one-hop propagation of the whistler wave packet along the duct. We found that, in contrast to the case of a wide duct, the group delay time in a rather narrow duct decreases as the eigenmode number increases. The results obtained are suggested for an explanation of some types of multi-component whistler signals.

Keywords. Electromagnetics (guided waves) – Magnetospheric physics (plasma waves and instabilities) – Radio science (waves in plasma)

1 Introduction

Problem of whistler mode wave propagation in magnetospheric ducts is very interesting in relation to many different problems of physics of the inner magnetosphere, for example, cyclotron wave particle interaction, interpretation of observations of whistler signals on satellites and ground stations, and others.

Magnetospheric ducts are the enhancements of the cold plasma density aligned with the geomagnetic field, which are extended between the Northern and Southern Hemispheres. Although the existence of such structures is known for a long time (Helliwell, 1965), very few direct satellite measurements of duct structures are made (Angerami, 1970; Sonwalkar et al., 1994), yet there are many evidences for the existence of ducts (see Singh et al. (1998) and references therein). These measurements demonstrate that parameters of magnetospheric ducts change over a very wide range: the radius of a duct can be from tens to hundreds of kilometers, and the density enhancement changes from fractions of ambient plasma density up to multiple increases in the case of a duct located outside the plasmasphere. It is clear that in the case when the duct radius is much greater than the characteristic wavelength the ray-tracing method is applicable (Strangeways, 1991, 1999). But if the duct radius is comparable with the wavelength it is necessary to perform a more strict analysis and to study the actual spatial structure and dispersion properties of waves propagating in a duct.

The possibility of whistler mode wave trapping in the density duct was studied in many papers, see, for example, Helliwell (1965); Karpman and Kaufman (1982); Strangeways (1991). It is well known that whistler-wave trapping is possible in the ducts of enhanced plasma density in the frequency range $\omega<\omega_B/2$, where ω_B is the electron gyrofrequency. But in this frequency range two dispersion branches exist: whistler and electrostatic mode, and the latter cannot be trapped by the duct. In the homogeneous plasma these waves are independent, but the inhomogeneity of the medium leads to a linear transformation between whistler and electrostatic modes. It results in the leakage of the whistler wave power from the duct. The efficiency of such a transformation is determined by the ratio of the inhomogeneity scale to the wave length (Karpman and Kaufman, 1982; Bell and Ngo, 1990).

In a general case it is not possible to find an analytical solution for the wave spatial structure in the duct with cylindrical geometry, therefore, numerical methods should be used (Laird and Nunn, 1975) for full-wave analysis. An analytical solution may be found only for a piecewise-constant distribution of plasma across the duct and a detailed analysis of this case was done by Kondratyev et al. (1996, 1999). However, in this case effective transformation between whistler and electrostatic waves is possible due to the sharp duct boundary, hence, this model is not applicable to the magnetosphere conditions, where the density distribution is rather smooth.

Another possibility for developing the mode theory of a whistler duct is to use the ray-tracing technique with the quasi-classical eigenmode condition, as in Laird (1992), for the plane-stratified duct model. In the frame of this model it is possible to study the propagation properties of

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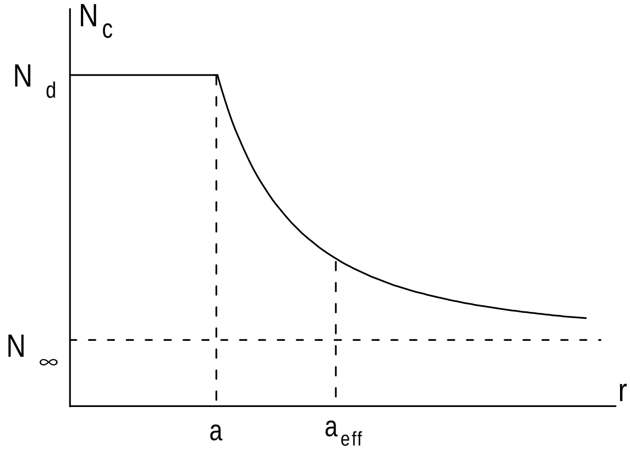

Fig. 1. Model plasma density distribution across the duct.

eigenmodes for a duct with smooth plasma distribution in the case where the duct radius is much greater than the characteristic whistler wavelength.

In this paper we consider a model of magnetospheric duct with smooth plasma density distribution. We present the profile of the plasma density distribution across the duct, for which an analytical solution for cylindrical duct eigenmodes exists. In contrast to the numerical consideration of Laird and Nunn (1975), where the duct is homogeneous along its axis, we consider the case of the duct whose parameters vary along its length. The peculiarities of whistler eigenmode propagation in the frame of this model are studied. The group delay time for eigenmode propagation along the duct is calculated. It is found that different eigenmodes have different propagation times and on the basis of this result an explanation of some types of multi-component whistler signals is suggested.

2 Magnetospheric duct eigenmodes

As a model of magnetospheric duct we consider a cylindrically symmetric plasma column filled with an inhomogeneous density distribution across its axis and located in an external magnetic field B_0 parallel to the axis.

At first, let us consider the homogeneous duct along its axis. We shall seek for the eigenmodes in the form of waves propagating along the duct and having some transverse structure:

$$\boldsymbol{E} = \boldsymbol{\Phi}(r)e^{i\omega t - ik_0 pz}\boldsymbol{H} = i\boldsymbol{\Psi}(r)e^{i\omega t - ik_0 pz}, \tag{1}$$

where $k_0 p = k_\parallel$ is the wave vector component along the duct axis z, r−coordinate across the duct, $k_0 = \omega/c$, and c is the speed of the light. Note that only axially symmetric modes are considered here.

Substituting these expressions into Maxwell's equations, we obtain two second-order ordinary differential equations for the electric-field components, Φ_φ and Φ_z, and expressions for other components of wave field, cf. Pasmanik and Trakhtengerts (2001). An explicit analytical solution for these equations may be found only for homogeneous plasma distribution or for a duct with a piecewise-constant radial density distribution. The results of rigorous analysis of this duct model are summarised in the book by Kondratyev et al. (1999). As it was mentioned in the Introduction, in this case an efficient transformation between whistler and electrostatic modes occurs, resulting in the leakage of wave power from the duct. Results from analytical (Karpman and Kaufman, 1982) and numerical (Laird and Nunn, 1975; Bell and Ngo, 1990) studies show that for a duct with a rather smooth boundary the efficiency of this transformation is small.

We shall consider the case of a rather smooth plasma distribution (i.e. $\left|N_c^{-1}\frac{dN_c}{dr}\right| \lesssim k_0 q_{el}$, where $N_c(r)$ is the plasma density, $k_0 q_{\mathrm{el}} = k_{\mathrm{el}\perp}$ is the characteristic value of the transverse wavevector of a quasi-electrostatic mode) and restrict our analysis to the low frequency waves ($\omega \ll \omega_B \ll \omega_p$, where ω_p is electron plasma frequency), whose propagation angle with respect to the magnetic field is not too large (i.e. $q \lesssim p$). Taking into account these simplifications we obtain the following equations for the wave field components (see Pasmanik and Trakhtengerts (2001) for details):

$$\begin{aligned}
&\Psi_r = ip\Phi_\varphi\ ,\ \Psi_\varphi = \frac{g}{p}\Phi_\varphi\ ,\ \Psi_z = \frac{1}{k_0 r}\frac{d}{dr}r\Phi_\varphi\ ,\\
&\Phi_r = i\frac{g}{p^2}\Phi_\varphi\ ,\ \Phi_z = -\frac{1}{k_0 p\eta}\frac{1}{r}\frac{d}{dr}rg\Phi_\varphi\ ,\\
&\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\Phi_\varphi\right) - \frac{\Phi_\varphi}{r^2} + k_0^2\left(\frac{g^2}{p^2} - p^2\right)\Phi_\varphi = 0,
\end{aligned} \tag{2}$$

where $g = -\omega_p^2\omega_B/\omega(\omega^2 - \omega_B^2)$ and $\eta = 1 - \omega_p^2/\omega^2$ are components of plasma dielectric permittivity tensor $\hat{\varepsilon}$, cf. Stix (1992). The solution of these equations corresponds to a whistler mode wave and the electrostatic mode is neglected.

The analytical solution of Eq. (2) for the eigenmode transverse structure can be found for the following plasma density distribution (Fig. 1):

$$N_c^2(r) = \begin{cases} N_d^2\ , & r \le a \\ (N_d^2 - N_\infty^2)\dfrac{a^2}{r^2} + N_\infty^2\ , & r > a, \end{cases} \tag{3}$$

where N_d and N_∞ are plasma densities inside and outside of the duct, and a is the radius of the duct core.

Let us note that under real conditions ducts are likely to have a bell-shaped (a Gaussian-like) profile. However, results obtained should not change much if we consider such types of density distribution across the duct. Use of different density profiles would result only in some quantitative changes in eigenmode properties. Concerning a discontinuity in the gradient, it does not play a sufficient role. Numerical analysis of a full set of Maxwell's equations using methods similar to that by Laird and Nunn (1975) and for density distribution (3) revealed that our simplified approach gives correct results. In particular, we have found that the transformation between the whistler and the electrostatic mode is negligible.

The solution for Φ_φ is represented in terms of the eigenmode series, and for distribution (3) it has the following form:

$$\Phi_{k\varphi}(r) = \begin{cases} B_k J_1(k_0 q_k r) & r \le a \\ C_k K_{\nu_k}(k_0 s_k r) & r > a, \end{cases} \tag{4}$$

where J and K are Bessel and Macdonald functions, respectively, $q_k^2 = g_d^2/p_k^2 - p_k^2$, $s_k^2 = p_k^2 - g_\infty^2/p_k^2$, $\nu_k^2 = 1 - k_0^2 a^2 (g_d^2 - g_\infty^2)/p_k^2$, $g_\alpha = g(N_\alpha)$, and k is the mode index. Values $k_0 q_k$ and $k_0 s_k$ represent the transverse component of the wave vector. The eigenvalues p_k and the corresponding values of q_k, s_k, ν_k, and C_k/B_k are found from the boundary condition of the continuity of the wave field components E_φ, H_φ, H_z at $r=a$:

$$\left.\frac{J_1(k_0 q_k r)}{(r J_1(k_0 q_k r))'} = \frac{K_{\nu_k}(k_0 s_k r)}{(r K_{\nu_k}(k_0 s_k r))'}\right|_{r=a} \tag{5}$$
$$C_k/B_k = J_1(k_0 q_k a)/K_{\nu_k}(k_0 s_k a)\ ,$$

where $(\ldots)'$ denotes the derivative $\frac{d}{dr}$.

Eigenmodes satisfy the following orthogonality conditions:

$$\begin{aligned} &\int_0^\infty r dr\ \frac{c}{4\pi} \left([\mathbf{\Phi}_k, \mathbf{\Psi}_{k'}^*]_z + [\mathbf{\Phi}_{k'}, \mathbf{\Psi}_k^*]_z\right) = S_k \cdot \delta_{k,k'} \\ &\int_0^\infty r dr\ \frac{1}{8\pi} \left(\frac{\partial \omega \hat{\varepsilon}}{\partial \omega} \mathbf{\Phi}_k \mathbf{\Phi}_{k'}^* + \mathbf{\Psi}_k \mathbf{\Psi}_{k'}^*\right) = W_k \cdot \delta_{k,k'}, \end{aligned} \tag{6}$$

where $\delta_{k,k'}$ is the Kronecker's symbol, and the asterisk denotes the complex conjugation. Here S_k and W_k have the meaning of the energy flux and energy density in the eigenmode, respectively.

To generalize these results to the case of duct whose parameters (magnetic field B_0, radius a, plasma density N_c) are changing slowly along the axis we use the method of slowly varying amplitudes, which gives us:

$$\begin{aligned} \boldsymbol{E}_k &= \ \mathbf{\Phi}_k(r,z) e^{i\omega t - ik_0 \int p_k dz} \\ \boldsymbol{H}_k &= i\mathbf{\Psi}_k(r,z) e^{i\omega t - ik_0 \int p_k dz} \end{aligned} \tag{7}$$

where $\mathbf{\Phi}_k(r,z)$, $\mathbf{\Psi}_k(r,z)$ and $p_k(z)$ are the solutions from the case of a homogeneous duct (see Eq. (4)) with parameters of the duct at given cross section z. Variation of the normalizing constant B_k along the z axis is determined from the energy flux conservation law: $S_k(z)$=const.

3 Eigenmodes propagation properties

In this section the results of numerical analysis of the properties of duct eigenmodes are presented. The following main characteristics of the eigenmode were considered: the parallel component of the wave vector p_k, the effective angle of wave propagation in the core of the duct $\alpha_k = \arctan(q_k/p_k)$, and the component of the group velocity parallel to the magnetic field $V_{G\|_k} = S_k/W_k$. These values characterize the local (relative to the coordinate along the duct axis) properties of the eigenmode propagation in the duct.

Another important parameter is the group delay time for the eigenmode propagation, which characterizes the propagation of a wave along the whole duct and is defined as

$$T_{G_k} = 2\int_0^l \frac{dz}{V_{G\|_k}}, \tag{8}$$

where l is the half-length of the magnetic field line.

The dependence of these values on the duct parameters, such as its radius a, the cold plasma density N_d and the density enhancement $\Delta = N_d/N_\infty$, is discussed below. It should be mentioned that for the chosen distribution of the cold plasma density across duct (3), more correct definitions for the duct width is needed. Let us define the characteristic radius (half-width) of a duct a_{eff} as the distance from the center to the point where the density enhancement $(N_\infty - N_c(r))$ decreases by a factor e (Fig. 1). This implies $a_{eff} \approx 2a$ for parameter values used below.

To characterize the width of the duct in comparison with the characteristic whistler wavelength we introduce the dimensionless parameter $k_{\|0} a$, where $k_{\|0} \approx k_0 \sqrt{g}$ is the value of the wave vector of a whistler propagating along the magnetic field in the uniform medium with parameters of the duct core. Our analysis showed that values $k_{\|0} a \gg 10$ correspond to the case of a wide duct, when the properties of eigenmodes are similar to the properties of waves propagating in a very smooth density distribution (i.e. to the case when a ray-tracing method is applicable), and valves $k_{\|0} a \lesssim 10$ correspond to the case of a narrow duct, when the properties of the eigenmode propagation are sufficiently different from the previous case. Since the value of $k_{\|0}$ depends on frequency let us also introduce parameter $\kappa = \omega_p/(2c)$ which is approximately equal to the value of $k_{\|0}$ for $\omega = \omega_B/4$, i.e. the value of $k_{\|0}$ at a characteristic whistler frequency.

The dependencies of eigenmode parameters on the frequency in the cases of wide and narrow ducts are presented in Fig. 2 and examples of the transverse structure of the eigenmodes are shown in Fig. 3. It should be noted that in our model the properties of eigenmodes depend on relative duct parameters, but not on their absolute values. The following set of dimensionless duct parameters was chosen: ω/ω_B, ω_{p_d}/ω_B, $\Delta_N \equiv N_d/N_\infty$, κa.

In the case of a wide duct eigenmodes propagate with a small angle to the duct axis (Fig. 2a) and the wave field exponentially decreases outside of the duct core (Fig. 3a). The properties of eigenmodes in this case are mostly determined by the plasma parameters in the inner part of the duct. With a decrease in the duct radius, the propagation angle increases and the eigenmode wave field sufficiently extends outside the duct core (Figs. 2b and 3b). Parameter $k_{\|0} a$ and thus, the effective width of the duct, increase with frequency. Consequently, the angle of eigenmode propagation and separation between eigenmodes (value of $p_k - p_{k+1}$) decrease.

An interesting feature of the eigenmode propagation in a narrow duct is the growth of the parallel component of

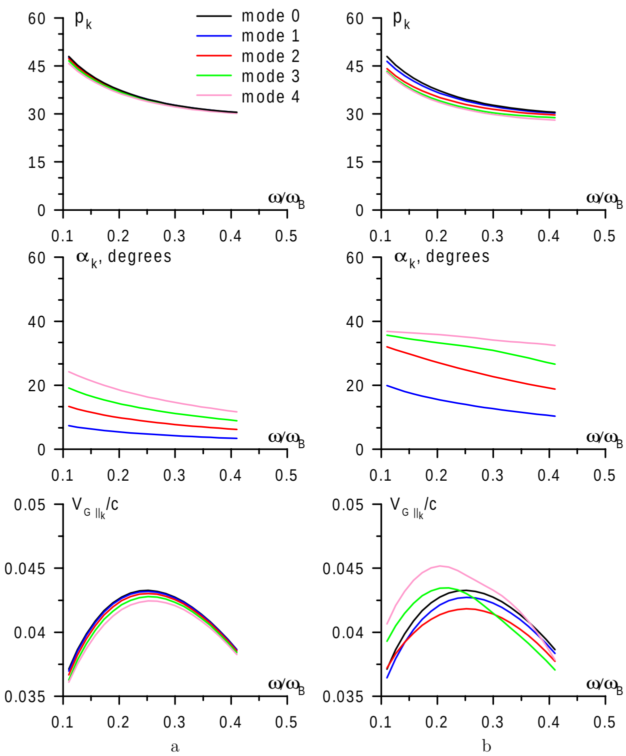

Fig. 2. Dependencies of eigenmode parameters on the frequency in the cases of wide (**a**) and narrow (**b**) ducts: a−κa=35, b−κa=10; eigenmode indexes are shown on the legend, index 0 corresponds to the whistler, propagating along the magnetic field in the plasma with parameters of the duct core; other parameters: ω_{p_d}/ω_B=15, Δ=1.25.

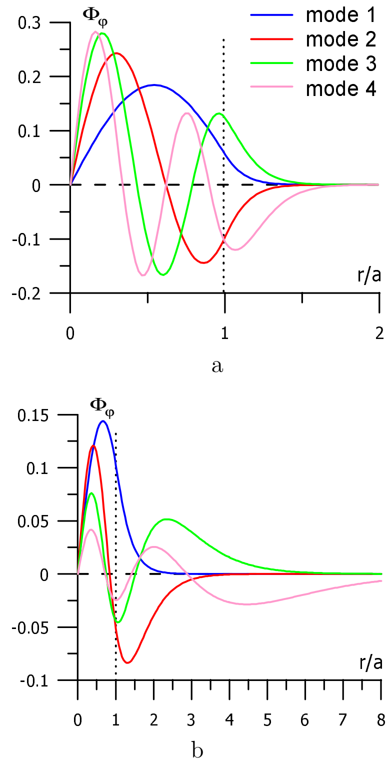

Fig. 3. Examples of the eigenmode transverse structure in the cases of wide (**a**) and narrow (**b**) ducts for the same parameter values as in Fig. 2 and ω/ω_{BL}=0.14. Eigenmode indexes are shown on the legend; the boundary of the duct core is marked by a dotted line.

the group velocity $V_{G\|_k}$ with an increase in a mode index and hence, a propagation angle. This occurs because in the outer part of the duct, where the plasma density is lower, the group velocity of a whistler wave is greater than in the inner part. With an increase in the eigenmode index, the transverse structure of the wave field expands further from the duct core, so the influence of the outer part of the duct on the wave propagation increases.

Dependencies of eigenmode parameters on the coordinate along the duct are shown in Fig. 4. To quantify the distance from the equatorial plane along the z coordinate we use the dimensionless parameter $B(z)/B_L$, where $B(z)$ is the local geomagnetic field value and $B_L=B(z=0)$ is the magnetic field at the equator. The relation between the duct radius and the magnetic field is defined as $a(z)=a_L\sqrt{B_L/B(z)}$ and the following distribution of the plasma density along the duct was considered $N_c(z)=N_{cL}B(z)/B_L$.

As we move away from the equatorial plane the duct becomes narrower (parameter $k_{\|0}a$ decreases), thus the angle of eigenmode propagation increases and the spatial structure of the eigenmode expands more outside of the inner part of the duct.

Let us now discuss the peculiarities of eigenmode propagation along the duct. We shall assume the existence of a wide frequency band whistler-wave source, which excites several eigenmodes simultaneously in the duct. As an example of such source the lightning discharge can be assumed. In this case a receiver, located in the conjugate footpoint of the duct, will register a set of discrete signals, each of them corresponding to one duct eigenmode. This feature of whistler propagation in a duct can be used to explain the multi-component whistler signals (see examples in the next section).

Examples of the group delay time for eigenmode propagation from the equatorial plane to the ionosphere at the magnetic shell L=4 are shown in Fig. 5. Three cases with different values of the duct radius are presented. As it was discussed above, with decrease in the duct radius the separation between eigenmodes increases. Thus, each component of the signal, corresponding to one of the duct eigenmodes, will be more pronounced in the case of a narrow duct (compare Figs. 5b and c). According to the dependence of the group velocity on the mode index discussed above, a signal corresponding to the highest mode arrives first and a signal corresponding to the first mode arrives last (see Fig. 5c). With an increase in duct radius the difference between the propagation time of the different eigenmodes decreases (see Fig. 5b). Finally, in the case of a rather wide duct, eigenmodes arrive in a sequence of increasing mode index, as it occurs in the case of a homogeneous medium (a decrease in

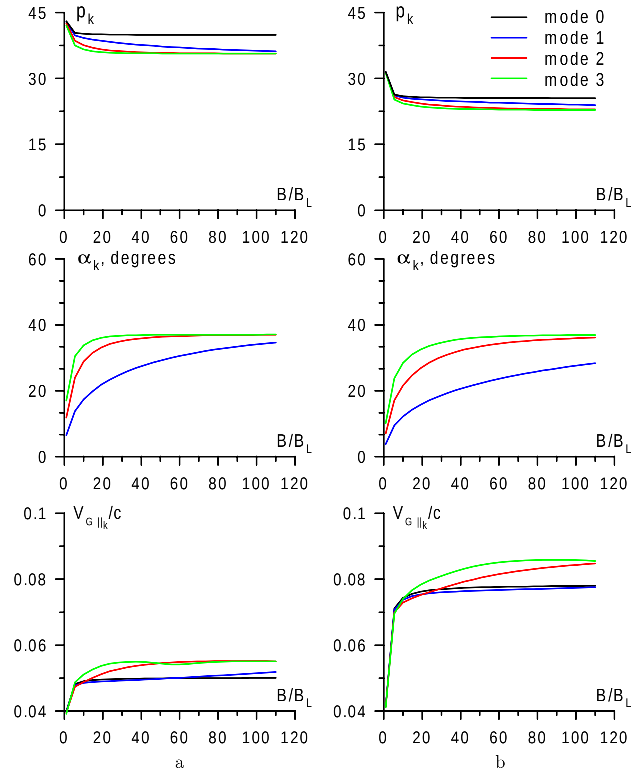

Fig. 4. Dependencies of eigenmode parameters on the coordinate along the duct (parameterized by magnetic field value) for different frequencies: a$-\omega/\omega_{BL}$=0.14, b$-\omega/\omega_{BL}$=0.35; the same legend as in Fig. 2 is used; other parameters: κa=5, $\omega_{p_d L}/\omega_{BL}$=15, $\omega_{p_d}/\omega_{p_\infty}$=1.25; subscript L refers to the values at the equatorial region.

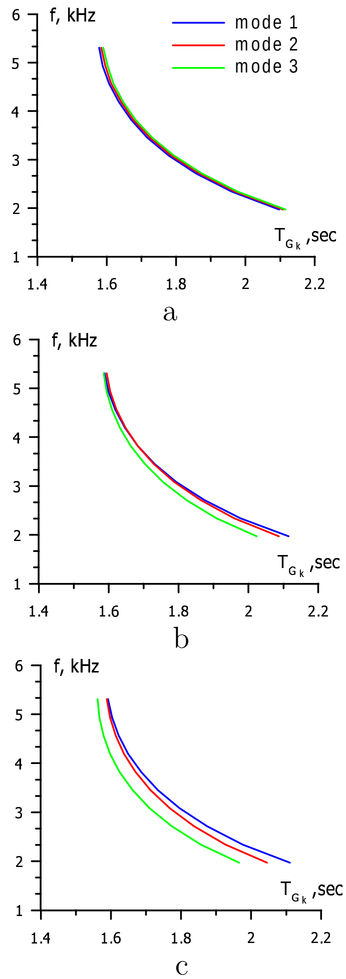

Fig. 5. The group delay time for eigenmode propagation from the equator to the ionosphere at magnetic shell L=4 (F_{BL}=13.5 kHz) for different values of the duct radius in the equatorial plane: a$-a_L\approx$90 km, b$-a_L\approx$30 km, c$-a_L\approx$20 km plasma density: $N_{dL}\approx$510 cm^{-3}, $N_{\infty L}\approx$330 cm^{-3} ($\Delta\approx$1.55) Eigenmode indexes are shown on the legend. Narrow duct case, plane (**c**): Separate components, corresponding to the particular eigenmode, are more pronounced in a signal; propagation time decreases with increase in the mode index. Wide duct case, plane (**a**): Components are less pronounced; propagation time increases with an increase in a mode index.

the group velocity with an increase in the propagation angle, see Fig. 5a).

Another possibility to explain multi-component signal is to assume an existence of several ducts located nearby, which are fed by one source (a lightning discharge) simultaneously. The difference in parameters of these ducts can lead to the formation of a multi-component signal, even in the case when only one eigenmode is excited in each duct. Below we consider such a case and assume that the first eigenmode (i.e. n=1) is excited. It should be noted that we use such an assumption to demonstrate this mechanism for multi-component whistler signal formation. In real conditions several eigenmodes can be excited in each duct, as in the case discussed above. Thus, both mechanisms can jointly act in the formation of a multi-component signal resulting in a more complex structure of the signal. However, accurate consideration of this case requires separate analysis including the solution of the eigenmode excitation problem by a source. This is out of scope of this paper.

The group delay time of the first eigenmode propagation in the ducts with different parameters is shown in Fig. 6. The case of the ducts with different density enhancement Δ (Δ=1.3, 1.4, 1.5), with the same values of the background plasma density N_∞, radius a, and located at the same L-shell, is shown in Fig. 6a. Two cases of ducts located at different L-shells (L=4, 4.05, 4.1), with the same values of Δ, a and N_∞, are shown in Figs. 6b and c. The last cases differ by the dependence of the background plasma density on the distance from the Earth, i.e. the dependence of the equatorial plasma density on the L-shell: in Fig. 6b the plasma density is constant over L ($N_{\infty_L}=const$), and in Fig. 6c the plasma density decreases with distance as $N_{\infty_L}\propto L^{-6}$.

An increase in the propagation time with an increase in Δ (Fig. 6a) is explained by the fact that whistler group velocity decreases with an increase in the plasma density in the duct. In the case of ducts at different L-shells the follow-

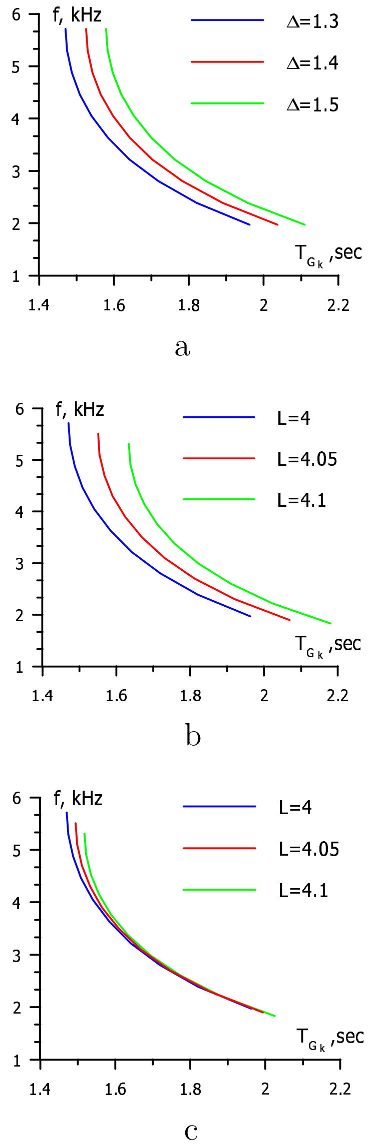

Fig. 6. The group delay time of the first eigenmode propagating from the eautor to the ionosphere in the different ducts:
a – ducts with differenct density enhancements $\Delta = N_{dL}/N_{\infty L}$ (L=4, $N_{\infty L} \approx 450\,\mathrm{cm}^{-3}$)
b – ducts at different magnetic shells L, plasma density is the same, (Δ=1.3, $N\infty L \approx 450\,\mathrm{cm}^{-3}$)
c – ducts at different magnetic shells L, plasma density $N_{\infty L} \propto L^{-3}$, ($\Delta$=1.3, $N_{\infty L} \approx 450\,\mathrm{cm}^{-3}$ at L=4).
Duct radius at the equator $a_L \approx 50$ km.

ing parameters determine the differences in the propagation time: the equatorial cyclotron frequency ω_{HL}, the magnetic field line length, and the relation between plasma- and cyclotron frequencies ω_{pL}/ω_{HL}. With an increase in L the equatorial cyclotron frequency decreases, thus in going over from dimentionless parameter ω/ω_{HL} to a real frequency value, we see that traces on the spectrogram move to lower frequencies. The length of the magnetic line, determining the path of wave propagation, increases, with L resulting in an increase in the propagation time. Only these two parameters are changing in the case presented in Fig. 6c, where the corresponding shift of eigenmode traces to the bottom-right direction occurs as L increases. In the case presented in Fig. 6b the value of ω_{pL}/ω_{HL} increases with L. This can be interpreted as an effective increase in the plasma density and it results in an additional increase in propagation time (see discussions above).

As one can see from Figs. 5 and 6 the cases of signals from one and several ducts have a significant difference: the distance between signals increases at lower frequencies in the first case, and decreases in the second case. This fact may be important for an interpretation of experimental data.

4 Conclusions

We have presented in this paper the model of a magnetospheric duct with a smooth plasma distribution across its axis. The great advantage of this model is that an analytical solution for the duct eigenmodes can be found. In contrast to the previous full-wave consideration by Laird and Nunn (1975), we studied the case of a duct whose parameters (the magnetic field, the radius, the cold plasma density) are varying along its axis. It allowed us to analyze the group delay time of the propagation of the different eigenmodes from a source (as, for example, lightning discharge) to the receiver (a satellite or a ground-based station in the geomagnetically conjugate point). We demonstrate that in the case of the source simultaneously exciting several duct eigenmodes, the receiver should register the set of discrete signals, each corresponding to one of the excited eigenmodes. The interval between the arrival of the different eigenmodes depends on the duct parameters, especially on its width; this interval is longer in the case of a narrow duct and smaller in the case of a wide duct. According to the results obtained this interval can vary from several up to hundreds of milliseconds. An interesting peculiarity of the decrease in the eigenmode propagation time with an increase in a mode index is found. This peculiarity is more pronounced at lower frequencies.

In the paper by Laird (1992) another method for whistler duct mode theory is used – a ray-tracing technique with a quasi-classical eigenmode condition in a plane-stratified duct model. This simplified consideration is very effective to study the propagation properties of whistlers in a rather wide duct, when its radius is much greater than characteristic whistler wavelength. This method demonstrates the same peculiarities of ducted whistler propagation as discussed above, in particular the decrease in propagation time with an increase in the mode index.

Unfortunately, it is not possible to perform the direct comparison of results obtained from our model with the results obtained by Laird (1992). The main difference between these two models is in a duct geometry and thus, in a different quantization condition for the eigenmodes. In the case of a plane geometry the eigenvalues are found from the condition that the phase change on the full ray path across the duct is a multiple of 2π:

$$2\int_{-a}^{a} k_{\perp} dx = 2\pi(n - \frac{1}{2}), \tag{9}$$

where $k_{\perp}$ and x are the wave vector component and the coordinate across the duct, respectively, and n is the mode index. In the case of a cylindrical geometry the condition for eigenvalues can be approximately written as

$$k_{\perp\, n} a = k_0 q_n a \approx \mu_n \tag{10}$$

in the case of a rather wide duct; here μ_n is the n-th root of the Bessel function $J_1(\mu_n)$=0.

It should also be mentioned that the mode theory, developed by Laird (1992), cannot be used to study the spatial structure of the wave field in the duct.

In the paper by Hamar et al. (1992) the matching filtering analysis was applied for VLF data recorded at the ground station Halley, Antarctica. The matching filtering technique allows one to obtain the dynamic spectrum with a very high resolution. Using this method authors have revealed that the whistler trace may consist of several fine structured traces. In particular examples presented in that paper such trace splitting was found to be on the order of 15–20 ms. Hamar et al. (1992) suggested that one of the possible mechanisms of this splitting is the multi-mode propagation of a signal in a duct. With the help of the model presented in this paper and using plasma parameters from Hamar et al. (1992) (L=4.566, N_{cL}=220 cm^{-3}) we can estimate other duct parameters from the condition that the splitting between the first and the second duct eigenmodes is 15–20 ms. We have found that it occurs for equatorial duct radius a_L∼50 km and density enhancement Δ∼1.35.

In another paper by Lichtenberger et al. (1996) the same type of analysis was applied for measurements on the Active (Intercosmos 24) satellite. Data presented in that paper demonstrate the doublets of whistlers, a signal consisting of several pairs of whistler traces. The separation between the signals in a doublet was about 80 ms. The fine structure of each of these traces (with approximately a 10 ms separation) was also revealed. Applying our model to this case with the available parameters: L=3.6, N_{cL}=400 cm^{-3}, Lichtenberger et al. (1996), we see that these doublets with 80 ms splitting could correspond to two eigenmodes of a duct with equatorial radius a_L∼35 km and density enhancement Δ∼1.45.

It would be very interesting to find experimental evidence for the obtained peculiarities of whistler propagation in a narrow duct, when the eigenmodes with higher indexes have a smaller group propagation time than the first eigenmode. For that to occur satellite measurements of several wave field components are required in order to determine the wave normal direction of different traces in the signal.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (grant No. 05-02-16459), the General Physics Division of the Russian Academy of Sciences (Program "Solar wind: Generation and Interaction with Earth and Other Planets"), and INTAS (grant No. 03-51-4132).

Topical Editor T. Pulkkinen thanks A. Smith for his help in evaluating this paper.

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